Chebyshev

Polynomial Interpolation

Math 426

University of Alaska Fairbanks

October 28, 2020

Polynomial Interpolation Error

$$f(x) = p(x) + f^{(n+1)}(\xi)\frac{\prod_{k=0}^{n}(x - x_k)}{(n+1)!}.$$

Keeping this small relies on keeping the product $(x - x_1) \cdot (x - x_n)$ small but also $f^{(n+1)}(\xi)$ small.

This can go wrong in ways that may surprise you.

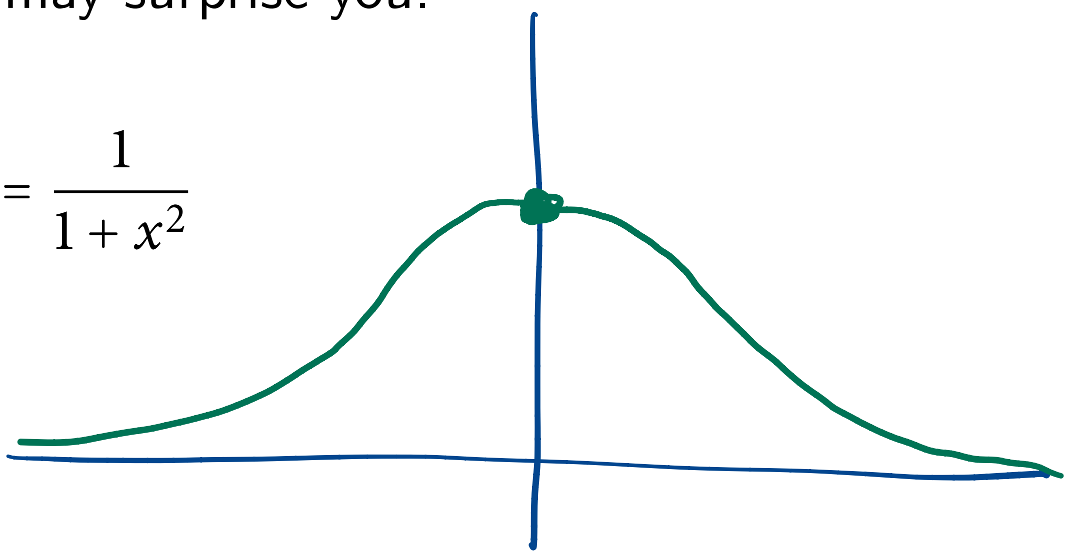
Polynomial Interpolation Error

$$f(x) = p(x) + f^{(n+1)}(\xi)\frac{\prod_{k=0}^{n}(x - x_k)}{(n+1)!}.$$

Keeping this small relies on keeping the product $(x - x_1) \cdot (x - x_n)$ small but also $f^{(n+1)}(\xi)$ small.

This can go wrong in ways that may surprise you.

$$f(x) = \frac{1}{1 + x^2}$$



Matlab Demo

What went wrong?

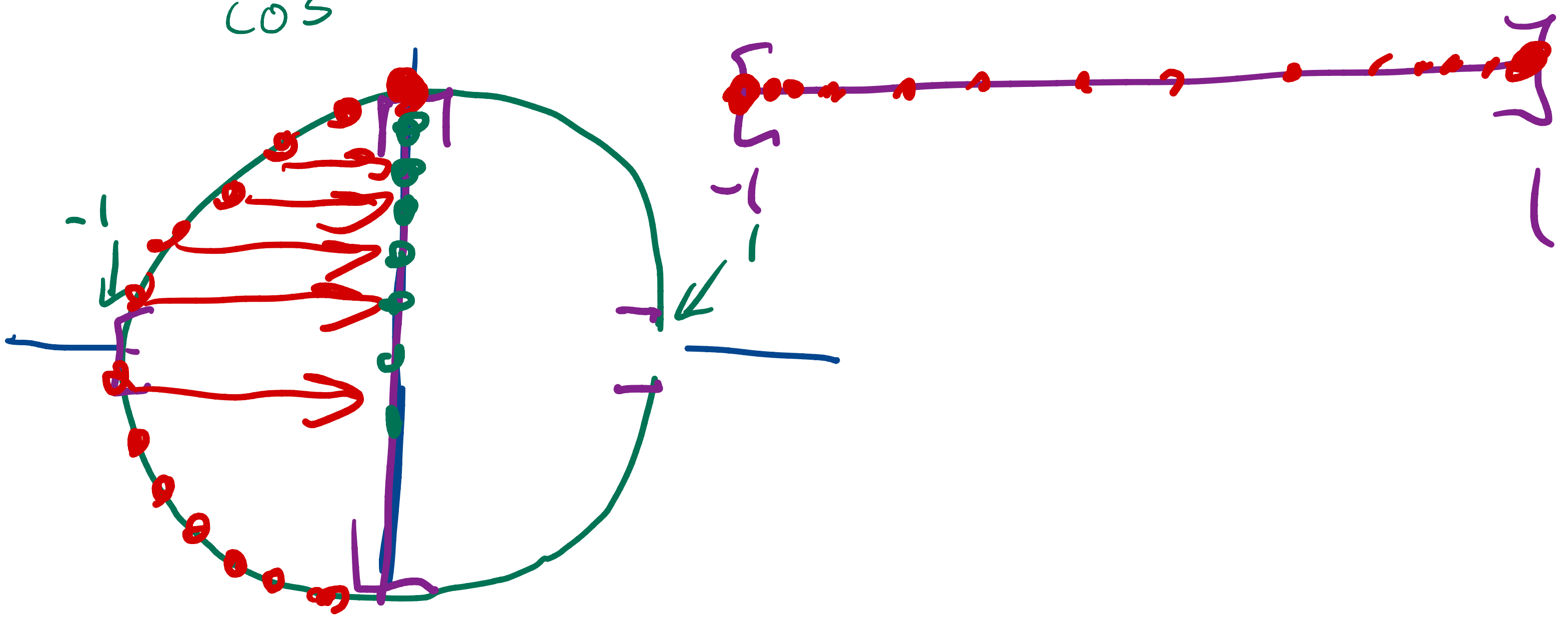
Chebyshev Polynomials

You don't always get to pick your sample points.

But if you can, there is a great choice.

On $[-1,1]$:

$$\hat{x}_j = \underset{\cos}{\cancel{\sin}}(\pi/2 + \overset{j}{\cancel{k}}(\pi/n)); \quad 0 \le j \le n$$



Chebyshev Polynomials

You don't always get to pick your sample points.

But if you can, there is a great choice.

On $[-1, 1]$:

$$\hat{x}_j = \sin(\pi/2 + k(\pi/n)); \quad 0 \le j \le n$$

Diagram:

On $[a, b]$, $x_j = a + (\hat{x}_j + 1)/2(b - a)$

Matlab Demo!