$$A\vec{x} = \vec{z}$$

$$A\tilde{x} = b + b_e$$

$$Ax = b$$

$$\|A\| \qquad \max_{\|x\|=1} \|Ax\|$$

$$\tilde{x} = \underbrace{A^{-1}b} + A^{-1}b_e$$

$$\tilde{x} = x + A^{-1}b_e$$

$$\|A^{-1}\| \qquad A^{-1}(b_e)$$

$$\tilde{x} - x = A^{-1}b_e$$

$$\frac{\|\tilde{x} - x\|}{\|x\|} = \frac{\|A^{-1}b_e\|}{\|x\|}$$

$$Ax = b \qquad \frac{\|b_e\|}{\|b\|}$$

$$\le \frac{\|A^{-1}\| \, \|b_e\|}{\|x\|}$$

$$\le \|A^{-1}\| \frac{\|b_e\|}{\|b\|} \frac{\|b\|}{\|x\|}$$

$$\le \|A^{-1}\| \frac{\|b_e\|}{\|b\|} \frac{\|A\| \, \|x\|}{\|x\|}$$

$$\frac{\|\tilde{x} - x\|}{\|x\|} \le \underbrace{\|A^{-1}\| \, \|A\|}_{\kappa} \frac{\|b_e\|}{\|b\|}$$

condition number

$\|A^{-1}\|$ how does absolute error scale?

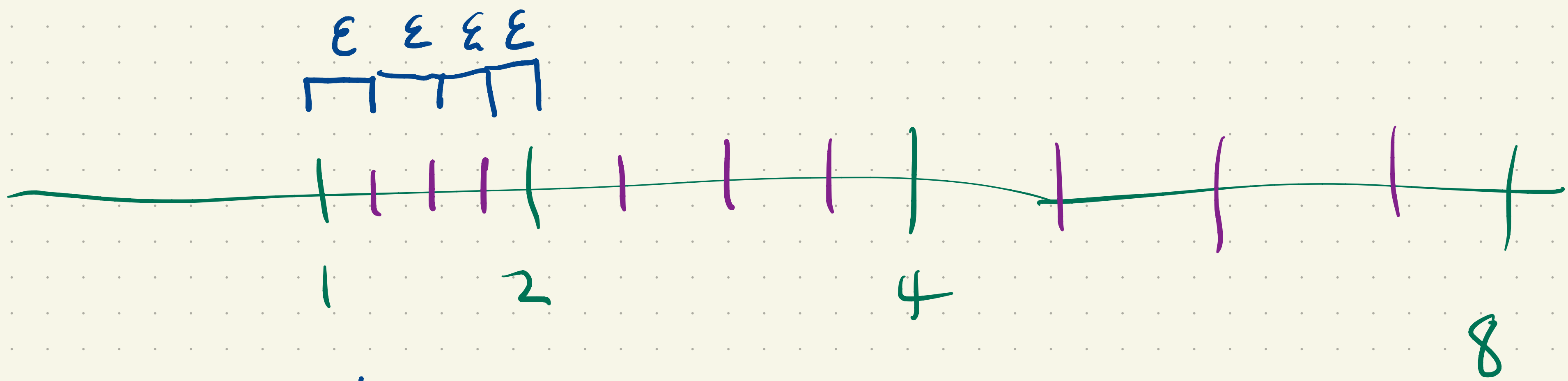
$\kappa \rightarrow$ how does relative error scale?

$\hookrightarrow$ relative errors at best $\sim 10^{-16}$

$\kappa \sim 10^{4}$

relative error in solution is $\sim 10^{-12}$

10^{13} 10^{-3}



$\frac{1}{2^2}$

$[2^k, 2^{k+1}]$ $2^k \cdot \varepsilon$ $\underbrace{|x - e|}$

$k=0$

x is closest to e $\qquad 2 \le e \le 4$

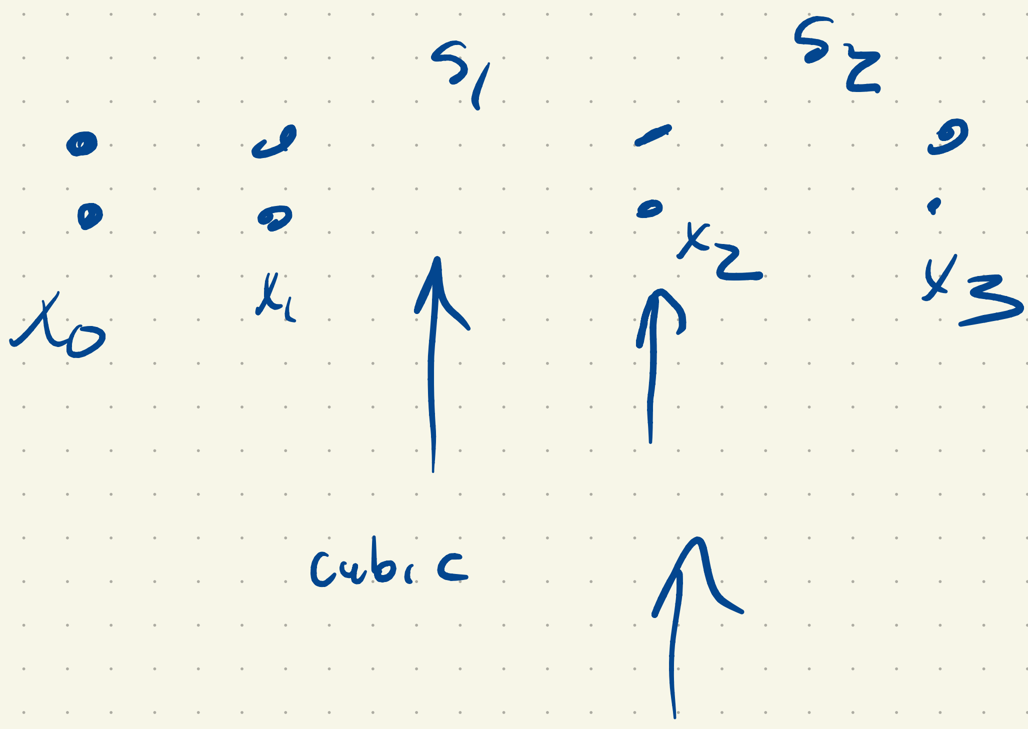
$$\langle p, q \rangle = \int_{-1}^{1} p(x)\, q(x)\, dx$$

1

$k = 5 \qquad 1, x, x^2, x^3, x \ldots, x^5$

$$p_5 \perp \{1, x, x^2, x^3, x^4\}$$

$x_0, \ldots, x_4$ roots of p_5



12 coeffs

6 values

2 derivs at s_1

2 derivs at s_2

$\to 8$

$$s_1(x_2) = s_2(x_2) = f(x_2)$$

$$s_1'(x_2) = s_2'(x_2)$$

$$s_1''(x_2) = s_2''(x_2)$$

10 conditions

give derivatives at ends.

give second derivatives at ends.

$3x^4 - 2x^3 + x - 7$

$(3x^3 - 2x^2 + 1)x - 7$

$((3x - 2)x^2 + 1)x - 7$

4 pm