The goal of this worksheet is for you to gain some insight on the phenomena that occur when you try to estimate the derivative of a function using a computer.

1. Let $f(x) = \sin(x)$. What is the exact value of $f'(\pi/3)$?

2. A standard approximation for the derivative of a function is

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \tag{1}$$

for some step size h. Use MATLAB to estimate $f'(\pi/3)$ using $h = 10^{-3}$.

3. More is better right? Use MATLAB to estimate $f'(\pi/3)$ using $h = 10^{-14}$. Did this do a better job than $h = 10^{-3}$?

4. What choice of h does the best job for approximating $f'(\pi/3)$? You may find it helpful to make a log-log plot of the error versus h. What is the smallest error you are able to make?

5. Repeat the previous exercise for $f(x) = \exp(x)$, approximating the derivative at $x = 1$. What value of h works best? What is the smallest error that you see?

6. A more egalitarian approach to computing the derivative would be

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}. \tag{2}$$

Give an intuitive reason why you expect this rule would do a better job.

7. Repeat problem 4 using this rule. What choice of h works best? What is the smallest error that you see?

8. Repeat problem 5 using this rule. What choice of h works best? What is the smallest error that you see?