The goal of this worksheet is for you to be able to write a Matlab code that performs LU factorization with partial pivoting.

There are some Matlab tricks that you will need to know in order to make your life easier.

Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}.$$

You should enter this matrix into Matlab.

1. What is the result of typing `A(1,:)`?

2. What is the result of typing `A(2,:)`?

3. What is the result of typing `A(:,2)`?

4. What is the result of typing `A(2,1:2)`?

5. What is the result of typing `A(2,2:3)`?

6. What is the result of typing `A([1,3,2],:)`?

7. What is the result of typing `B=A; B(2,:)=[-1,3,5]`?

8. What does `x=A(2,:)` do?

9. At the Matlab command line, type the following:

```
B = [1 2 3; 4 5 6; 7 8 0 ];
B(2,:) = B(2,:) -4*B(1,:)
```

How is the new value of `B` related to the old value?

10. In your group, write a function `[L,U]=mylu(A)` that performs LU decomposition **without** pivoting. You may already have code that you have prepared for the upcoming homework and are welcome to use it. Verify that your function works by testing it on

$$A = \begin{pmatrix} -4 & 3 & 3 \\ 20 & -13 & -14 \\ -16 & 8 & 8 \end{pmatrix}.$$

You already performed the LU decomposition of this by hand on the previous worksheet. And you should be able to compare LU versus A in Matlab and verify that they are the same up to rounding.

11. Going back to Matlab, what is the result of:

```
x=[ 4, -19, 7, -1]
v=max(x)
[v2,i]=max(x)
```

12. What is the result of:

```
x=[ 4, -19, 7, -1]
v=max(abs(x))
[v2,i]=max(abs(x))
```

13. What is the result of:

```
A=[1 2 3; 3 5 6; 7 8 0]
[v,i]=max(abs(A(1:3,1)))
```

14. What is the result of:

```
A=[1 2 3; 3 5 6; 7 8 0]
[v,i]=max(abs(A(2:3,2)))
```

Pay close attention to `i`.

15. What is the result of:

```
A=[1 2 3; 3 5 6; 7 8 0]
tmp = A(1,:);
A(1,:) = A(2,:);
A(2,:) = A(1,:);
A
```

16. Suppose

```
L=[1 0 0 0; 2 1 0 0; 3 4 1 0; 5 6 0 1 ]
```

Find Matlab commands that exchange the portions of rows 3 and 4 to the left of the diagonal (so the entries $[3,4]$ should exchange with the entries $[5,6]$).

17. Suppose instead

```
L=[1 0 0 0; 2 1 0 0; 3 0 1 0; 5 0 0 1 ]
```

Find commands that exchange the portions of rows 2 and 4 to the left of the diagonal. Stop and talk to me before proceeding.

18. Consider the matrix

$$A = \begin{pmatrix} 15 & -16 & 20 & 72 \\ 6 & -5 & 4 & 20 \\ 9 & -10 & 12 & 51 \\ 3 & -2 & 1 & 7 \end{pmatrix}$$

Verify that the matrices

$$P = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}; \qquad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ 5 & 6 & 0 & 1 \end{pmatrix}; \qquad U = \begin{pmatrix} 3 & -2 & 1 & 7 \\ 0 & -1 & 2 & 6 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 3 & 1 \end{pmatrix};$$

satisfy

$$PA = LU.$$

At the Matlab command line, perform a step of Gaussian elimination with partial pivoting to clear column 3. What are the new matrics P, L and U? Verify that $PA = LU$ still.

19. With your partner, write a function `[P,L,U]=myplu(A)` that performs LU decomposition with partial pivoting and returns a permutation matrix P, a lower-triangular matrix L, and an upper triangular matrix U such that $PA = LU$. Test that your function works by verifying it against your solution to Part B of the previous worksheet.

20. Use your function `myplu` along with your function `usolve` written for last week's homework and the function `lsolve` on the course web page to solve the system $Ax = b$ where

$$b = [21, -101, 72]^T.$$

21. If you get here and are bored, modify your `myplu` function so that instead of a full permutation matrix P it returns a vector where the entry for each row contains the column index where a 1 lies. For example,

$$P = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

is represented by $[4, 2, 3, 1]^T$.