Exercise 7.9:

Exercise 7.10:

Exercise 7.14:

Supplemental 1: Let

$$A = \begin{pmatrix} 1 & -1 & 0 & \alpha-\beta & \beta \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}; \qquad \mathbf{b} = \begin{pmatrix} \alpha \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

a) Show that that for any choice of numbers α and β, the solution of $A\mathbf{x} = \mathbf{b} = (1, 1, 1, 1, 1)^T$.

b) This is an upper triangular matrix! For $\alpha = 0.1$ and $\beta = 10^1, 10^2, \ldots, 10^{12}$ solve $A\mathbf{x} = \mathbf{b}$ using your `usolve` code. Present a table of $||\mathbf{x} - \hat{\mathbf{x}}||_\infty$

c) Present a table of the ∞ norm condition numbers of the matrices A from the previous problem.

d) Discuss the relationship between parts (b) and (c).

Supplemental 2: Suppose you have data points $(1, y_1), \ldots, (n, y_n)$ and that the points $(k, \log(y_k))$ all lie on a line with positive slope. Show that there are constants $C > 0$ and $\alpha > 1$ such that

$$y_k = C\alpha^k$$

Supplemental 3: We will shortly be seeing the Vandermonde matrices, which show up when doing polynomial interpolation. So, they appear naturally in the real world, and the point of this exercise is to characterize just how awfully their condition number grows as the size of the matrix grows.

Given a vector $\mathbf{x} = (x_0, x_1, \ldots, x_n)^T$, the $(n + 1) \times (n + 1)$ Vandermode matrix associated with $\mathbf{x}$ is defined on page 181 in your text. You can create one in matlab with the command `vander(x)`.

1. For $n = 1, 2, \ldots, 20$, let $\mathbf{x} = (0, 1/n, 2/n, \ldots, 1)$, and let κ_n be the 2-norm condition number of the Vandermonde matrix associated with $\mathbf{x}$. Make a plot of $\log(\kappa_n)$ versus n.

2. If everything has gone well, your plot will look like a straight line! Use a least squares method to find m and b such that

$$\log(\kappa_n) \approx mn + b$$

Then plot your line on the same graph as in part (b).

For full credit, you must show the matlab commands used to obtain m and b.

3. Find constants C and α such that

$$\kappa_n \approx C\alpha^n$$