

The exam will cover all material taught from Chapters 2, 4, 5, 6, 7, 8, 9, and 10. There will be an emphasis on material after Chapter 7, section 2.

Here are some ideas of things you should be able to do on the exam, beyond those already listed on the midterm study sheet, which you should also review.

If a topic from the covered material isn't here, that doesn't mean it won't be on the exam.

You should also look over your homework problems for ideas of things to study. If I assigned a question, I did so because I thought the problem was important. If I wrote the question myself, I must have thought it was especially important! The questions on the exam will generally be closely inspired by problems you saw on homework.

The exam is closed book. No calculators will be allowed. You may bring into the exam a half of a letter-sized piece of paper with notes written on one side.

- What is the condition number of a matrix? Given a 2×2 matrix, could you compute its condition number (with respect to the 1-norm)? Be able to use the condition number to estimate the error in the solution of a linear system.
- What is the difference between relative error and absolute error? What's a relative condition number? What's an absolute condition number? What's a relative condition number good for, anyway?
- Know the Vandermonde, Lagrange and Newton methods for finding the interpolating polynomial. You should be able to set up the matrix that solves for the coefficients of the Vandermonde and Newton forms.
- Know the error formula for polynomial interpolation and be able to apply it to estimate error. (E.g. Homework 11, Supplemental 1)
- Ditto for piecewise polynomial interpolation (page 198).
- Know the definition of a cubic spline. What are not-a-knot boundary conditions?
- Be able to construct a quadratic spline given knot data.
- What are the conditions that define a cubic spline on the interior nodes? What about at the boundary?
- Be able to perform a few iterations of Richardson extrapolation, as in the worksheet from class. What is the purpose of Richardson extrapolation, anyway?
- Given a discrete approximation of a derivative, be able to use Taylor's theorem to compute its order of accuracy. (e.g. Text problem 9.5)
- Given an $O(h^k)$ method for computing a derivative, suppose the roundoff error goes like ϵ/h^ℓ . Here, k and ℓ are both positive. What would be a good choice of h to minimize error?

- Given $n + 1$ sample points x_k in $[a, b]$, be able to set up the linear system to solve for weights A_k such that

$$\int_a^b p(x) dx = \sum_{k=0}^n A_k p(x_k)$$

for every polynomial of degree $\leq n$. (Note: if the x_k 's are equally spaced, this is what is known as a Newton-Coates formula).

- Given an error formula for the trapezoidal or Simpson's rule (page 229 formula 10.1, or page 232 just after formula 10.4), be able to estimate the error in an integral approximation. E.g., Example 10.1.2.
- Suppose I give you sample points and weights on $[-1, 1]$. How to you obtain corresponding sample points and weights on any $[a, b]$? How to you obtain a corresponding composite integration rule on n subintervals?
- We saw the phenomenon that some choices of quadrature sample points (on a single interval) are better than others.

What was the choice that defined the sample points for Clenshaw-Curtis integration? Why would one expect these to be good?

What was the choice that led to Gauss-Legendre integration? Given $n + 1$ sample points, what order of approximation does Gauss-Legendre integration have?

- Know how to implement the first few steps of the Gram-Schmidt algorithm.