

Bisection

Math 426

University of Alaska Fairbanks

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- ▶ $x^6 + 12x^2 + 3x = 4$

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1. Find an approximate solution x_{approx} so that $f(x_{\text{approx}}) \approx c$.
2. Find an estimate for the size of the **error**

$$\text{error} = |x_{\text{exact}} - x_{\text{approx}}|$$

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So we can always transform the equation so that $c = 0$. We'll use F for the name of the function. A solution of $F(x) = 0$ is call a **root** of F .

Idea of Bisection

Suppose we know numbers a and b with $a < b$ and

$$F(a) < 0$$

$$F(b) > 0$$

Then there should be a c somewhere in the middle so that $F(c) = 0$.