Continuity:

$$f : A \to \mathbb{R} \qquad A \subseteq \mathbb{R}$$

We say that f is continuous at $a \in A$

if for every $\varepsilon > 0$ there exists $\delta > 0$

such that if $x \in A$ and $|x-a| < \delta$

$0<$

then $|f(x) - f(a)| < \varepsilon$.

$0 < |x-a| < \delta$

$$\lim_{x \to a} f(x) = f(a)$$

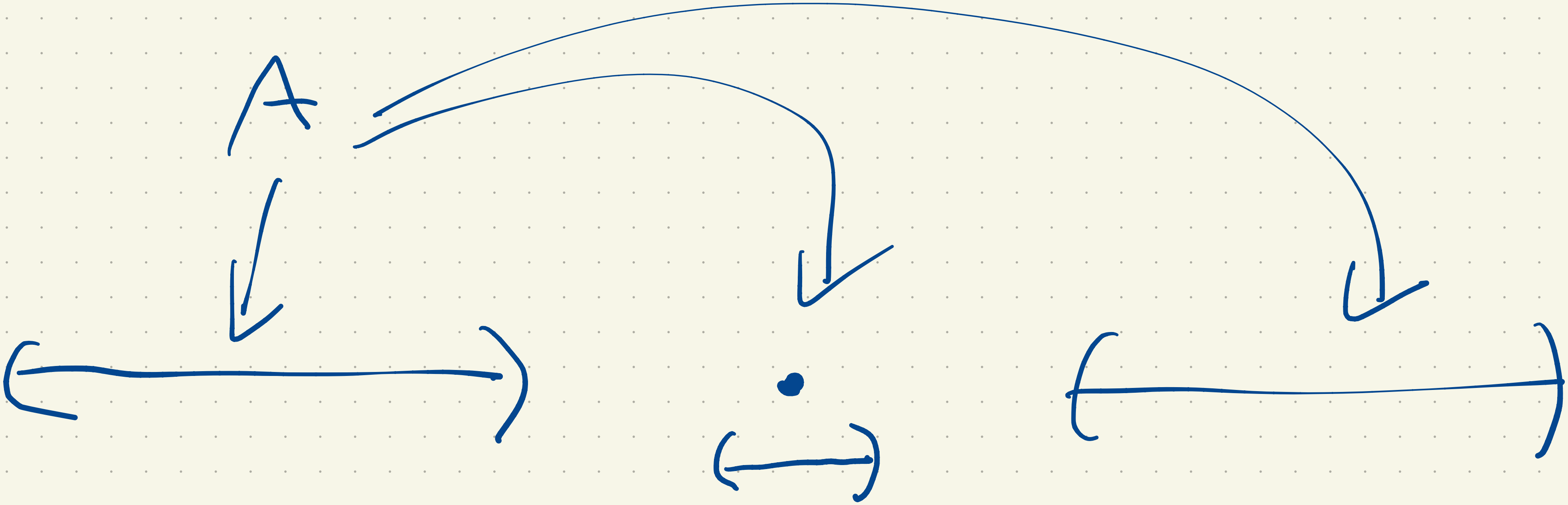
$\lim_{x \to c} f(x)$ needs c is a limit point of A

$$A = (0,1)$$

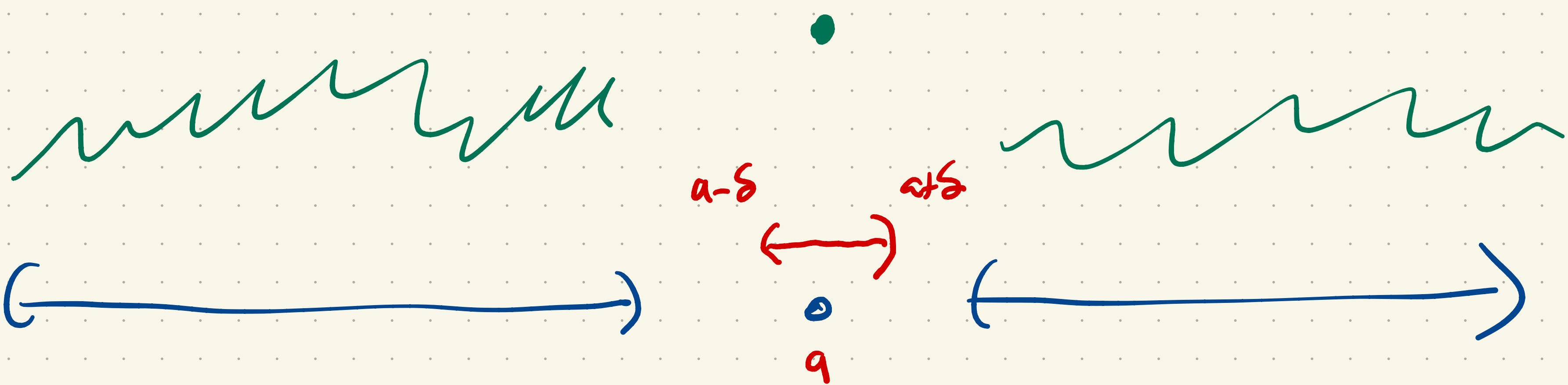
Prop: Suppose $a \in A$, and a is a limit point of A.

Then f is continuous at a if and only if

$$\lim_{x \to a} f(x) = f(a).$$



At isolated points, a function is always continuous.



$$V_\delta(a) \cap A = \{a\}$$

If $x \in A$ and $|x-a| < \delta$

then $x = a$.

$$\underbrace{|f(x) - f(a)|}_{0} < \varepsilon$$

Prop: (Sequential Characterization of Continuity)

Suppose $f: A \to \mathbb{R}$, and $a \in A$.

Then f is continuous at a if and only if whenever (x_n) is a sequence in A, $x_n \to a$ then $f(x_n) \longrightarrow f(a)$.

[Reprove as per functional limits.]

$f, g : A \to \mathbb{R}$

both continuous at $a \in A$.

$f + g$ is continuous at a.

Suppose (x_n) is a sequence in A with

$x_n \to a$. Then $f(x_n) \to f(a)$
$g(x_n) \to g(a)$

But then $f(x_n) + g(x_n) \to f(a) + g(a)$