Preamble: There is a total of **51** points on this exam. You have one hour to complete the exam. Please write out your proofs and your statements of theorems and definitions as clearly as you can. In grading, however, some consideration will be made for the exam's time constraints. Good luck!

1. [12 points] Define the following.

a) A limit point of a set.

b) $\lim_{x\to c} f(x) = L$.

c) $f : A \to \mathbb{R}$ is continuous at $c \in A$.

d) A closed set.

e) f is uniformly continuous.

f) $\lim_{x\to\infty} f(x) = L$.

2. [9 points] State the following.

a) The Sequential Characterization of Continuity

b) The Intermediate Value Theorem

c) The Extreme Value Theorem.

3. [10 points] Use the Sequential Characterization of Functional limits to show that if $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then $\lim_{x\to c} f(x)g(x) = LM$.

4. [10 points] Use the ϵ-δ definition of continuity to show that $f(x) = x^5$ is continuous at $x = 0$.

5. [5 points] Show, from the definition, that if $x > 3$ then x is not a limit point of $[2,3]$. Then show that $[2,3]$ is a closed set.

6. [5 points] For each of the following, give an example (or state why the request is impossible.)

a) A continuous function on $[0,1]$ that takes on exactly two values.

b) A continuous function $f : [0,1] \to \mathbb{R}$ such that $f([0,1]) \supseteq \mathbb{Z}$.

c) A continuous function $f : [0,1] \to \mathbb{R}$ such that $f([0,1)) = (4,6)$.

d) A continuous function on $(0,1]$ that attains neither a maximum nor a minimum.

e) A bounded function on $(0,1]$ that is not uniformly continuous.