Preamble: There is a total of **60** points on this exam. You have one hour to complete the exam. Please write out your proofs and your statements of theorems and definitions as clearly as you can. In grading, however, some consideration will be made for the exam's time constraints. Good luck!

1. [16 points] State the following theorems (or axioms)

a) Axiom of Completeness

b) Bolzano Weierstrass Theorem

c) Nested Interval Property of $\mathbb{R}$

d) Archemedian Property of $\mathbb{R}$

e) Cauchy Criterion (for series)

f) Alternating Series Test

g) The Triangle Inequality

h) The Squeeze Theorem

2. [14 points] Define the following.

a) $\lim_{n\to\infty} a_n = L$.

b) $\sum_{k=1}^{\infty} a_k = S$.

c) The **infimum** of a set $A \subseteq \mathbb{R}$; define any new terms you introduce.

d) A sequence (a_k) is **bounded**.

e) A **finite** set.

f) A monotone increasing sequence.

g) The sequence (a_n) is **Cauchy**.

3. [10 points] Use the definition of convergence to compute $\lim_{n\to\infty}(1+\frac{1}{n})^2$. You may use the fact that for every positive real number x there exists $n \in \mathbb{N}$ such that $1/n < x$.

4. [8 points] Let $A = \{-1/n : n \in \mathbb{N}\}$. Prove that $\sup A = 0$.

5. [6 points] Show that a convergent sequence is bounded.

6. [6 points] Suppose $0 \le a_k \le 1$ for each k and that $\sum_{k=1}^{\infty} a_k$ converges. Show that $\sum_{k=1}^{\infty}(a_k)^3$ converges.

Extra credit: [2 points] Is the restriction $a_k \le 1$ necessary?