Exercise 2.4.5 (Modified, with hints!): Suppose $x_1 = 2$ and define

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right).$$

1. Show that $x_n \geq 0$ for all n.
2. Show that if $a > 0$ then $a + \frac{1}{a} \geq 2$. Hint: $(a-1)^2 \geq 0$. [Your proof should highlight the part where you use the hypothesis $a > 0$.]
3. Show that if $b \neq 0$ then $b^2 + 4/b^2 \geq 4$. Hint: Use the previous item!
4. Show that $x_n^2 \geq 2$ for all n. Hint: Use the previous item!
5. Show that $x_n \geq x_{n-1}$ for all n. Hint: Use the previous item!
6. Show that the sequence converges to a limit L.
7. Show that $L \neq 0$. Hint: If $x_n \to 0$ then $x_n^2 \to 0$.
8. Show that $L^2 = 2$. Hint: $\lim x_{n+1} = \lim x_n$.

Exercise 2.5.5:

Exercise 2.5.6:

Exercise 2.5.7:

Exercise 2.6.2:

Exercise 2.6.7 (b):