e.g. A rocket accelerates for 10 years (proper!) at 10 m/s.

How much rest time elapses?
How far does it travel w.r.t. the rest frame.

$$K = 10 \text{ m/s}$$

$$\alpha = \frac{c^2}{K}\begin{bmatrix}\sinh\left(\frac{K}{c}\tau\right)\\ \cosh\left(\frac{K}{c}\tau\right)\end{bmatrix}$$

1 year $\approx 3\times10^7$ seconds $c \approx 3\times10^8$ m/s =

$\tau = 10$ years $\approx 3\times10^8$ seconds $\frac{c}{10 \text{ m/s}^2} = 3\times10^7 s \approx 1$ year

$$\frac{K\tau}{c} = \frac{10}{c}\cdot 3\times10^8 = \frac{3\times10^9}{3\times10^8} = 10$$

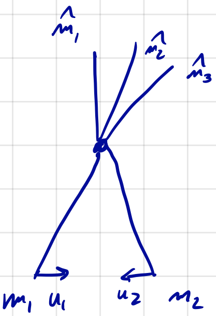
rest frame time elapsed $\frac{c}{10}\sinh(10) = 11\,000$ years

distance $c\cdot\frac{c}{10}\left[\cosh(10)-1\right] \approx c\frac{c}{10}\sinh(10) = 11\,000$ l.y.

11 kly

MY center 27 Kly, edge 100 Kly andromeda 2 500 k ly

Recall from Phys 101



Rules 1) conservation of mass

$$\sum_{i=1}^{n} m_i = \sum_{j=1}^{k} \hat{m}_j$$

2) Linear momentum: $m_i \cdot u_i$

Conservation of linear momentum

$$\underbrace{\sum_{i=1}^{n} m_i u_i}_{\text{total momentum prior}} = \underbrace{\sum_{j=1}^{k} \hat{m}_i \hat{u}_i}_{\text{and after}}$$

3) $$\sum_{i=1}^{n} \frac{1}{2} m_i |u_i|^2 = \sum_{j=1}^{k} \hat{m}_j \, |\hat{u}_j|^2$$

Conservation of energy.

(it may be there is other energy involved; thermal, chemical)

$$E_I + \sum \frac{m_i}{2}|u_i|^2 = \hat{E}_I + \sum \frac{\hat{m}_j}{2}|\hat{u}_j|^2$$

2) is a consequence of Newton 2, 3

$$\frac{d}{dt} p_1 = F_{21} \qquad \frac{d}{dt} p_2 = F_{12} = -F_{21}$$

$$\frac{d}{dt}(p_1 + p_2) = 0$$

We'll assume other observers agree on $m_i, \hat{m}_i$ and $E_I, \hat{E}_I$.

1), 2), 3) in one Galilean ~~frame~~ frame $\Rightarrow$ in all:

$u_i \longrightarrow u_i - v$ in a frame boosted by vel v.

$$\begin{aligned}
\sum m_i (u_i - v) &= \sum m_i u_i - \left(\sum m_i\right) v \\
&= \sum_j \hat{m}_j \hat{u}_j - \left(\sum \hat{m}_j\right) v \\
&= \sum_j m_j (\hat{u}_j - v)
\end{aligned}$$

$$\begin{aligned}
\sum \tfrac{1}{2} m_i |u_i - v|^2 &= \sum \frac{m_i}{2}|u_i|^2 - \sum m_i u_i \cdot v \; \sum_i \frac{1}{2} m_i |v|^2 \\
&= \sum \frac{\hat{m}_j}{2} |u_j|^2 - \left(\sum \hat{m}_j \hat{u}_j\right) \cdot v + \left(\sum_j \tfrac{1}{2} \hat{m}_j\right) |v|^2
\end{aligned}$$

$$= \sum \frac{1}{2} \hat{m}_j |u_j - v|^2$$

Exercise: If 3) holds in every galilean frame then so do 1) and 2)

Conservation of energy is bedrock.

These laws are not compatible with SR.

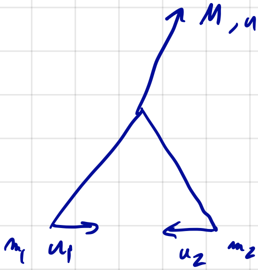
Recall, for a boosted observer velocity u becomes

$$\frac{u-v}{1-\frac{uv}{c^2}}$$

$$\begin{bmatrix} C & -S \\ -S & C \end{bmatrix} \begin{bmatrix} c \\ u \end{bmatrix} = \begin{matrix} Cc - Su \\ -Sc + Cu \end{matrix}$$

$$\frac{-Sc + Cu}{C - Su/c}$$

$$\frac{u-v}{1-\frac{v}{c}\frac{u}{c}}$$



$$\mathcal{M} = m_1 + m_2$$

$$m_1 u_1 + m_2 u_2 = M u$$

$$u = \left(\frac{m_1}{M} u_1\right) + \left(\frac{m_2}{M}\right) u_2$$

$m_1 = m_2 = m$ $\qquad u = \frac{1}{2}(u_1 + u_2)$

E.g. $u_2 = -u_1$

$u_1 \qquad u_2 = -u_1$

Now boost to frame with vel u_1

$$\hat{u}_1 = 0 \qquad \hat{u}_2 = \frac{u_2 - u_1}{1 - \frac{u_1 u_2}{c^2}} = \frac{-2u_1}{1 + \frac{u_1^2}{c^2}}$$

$$\hat{u} = \frac{(u_1 + u_2)/2 - u_1}{1 - \frac{u_1(u_1+u_2)}{2c^2}} = \frac{-u_1}{1 - 0} = -u_1$$

$$\frac{\hat{u}_1 + \hat{u}_2}{2} = \frac{-u_1}{1 + \frac{u_1^2}{c^2}} \neq -u_1 = \hat{u}$$ unless $u_1 = 0$.

Instead: $P_i = m_i V_i$ → 4-velocity of particle i

↑ mass of particle i.

4-momentum of particle i.

Note: two masses in physics:

1) Inertial

$$\frac{d}{dt}(mv) = F$$

2) Gravitational $\quad F = -\frac{GmM}{r^2}$

The mass here is (almost) inertial, not gravitational.

We call it the <u>rest</u> <u>mass</u> of the particle, and it is an intrinsic quantity agreed upon by all observers

$$P_i = m_i \, \gamma(v) \begin{bmatrix} c \\ V_i \end{bmatrix}$$

New law: 4 momentum is preserved

$$\sum_i P_i = \sum_j \hat{P}_j$$

a) time component $\quad c \sum m_i \gamma(v_i) = c \sum \hat{m}_j \gamma(\hat{v}_j)$

b) space $\quad \sum m_i \gamma(v_i) V_i = \sum \hat{m}_j \gamma(\hat{v}_j) \hat{V}_j$

interpretation 1) $\quad m_i \gamma(v_i)$ is the observed mass of particle i.

This is a) conservation of mass

b) conservation of momentum

$m \gamma(v) = m_I$

$m_I V \longrightarrow$ 3- momentum.

Interpretation 2)

$$\gamma(v) = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} = 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + O\left(\left(\frac{v}{c}\right)^4\right)$$

$$cm\,\gamma(v) = cm + \frac{1}{2}\frac{m}{c}v^2 + O\left(\left(\frac{v}{c}\right)^4\right)c$$

Multiply by one more c

$$\underbrace{mc^2}_{\text{rest energy of the particle, proportional to rest mass}} + \underbrace{\frac{1}{2}mv^2}_{\text{classical kinetic energy}} + O\left(\left(\frac{v}{c}\right)^4\right)mc^2$$

$$p = \begin{bmatrix} p^0 \\ p^1 \\ p^2 \\ p^3 \end{bmatrix}$$

cp^0 is interpreted as the observed energy of the particle (rest + kinetic) in this frame.

$cp^0 = c^2\gamma(v)m$, minimized at mc^2 at $v=0$.

justifies the name "rest energy"

We will start representing observers in spacetime by timelike vectors of length c.

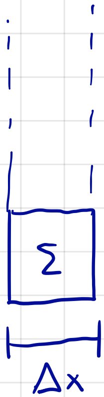
What is the energy measured by $\underset{\sim}{U}$?

In a frame where $U = \begin{bmatrix} c \\ 0 \end{bmatrix}$ it is $cP^0 = c\,g(P,U)$.

In another frame $g(P,U) = g(\hat{P}, \hat{U})$.

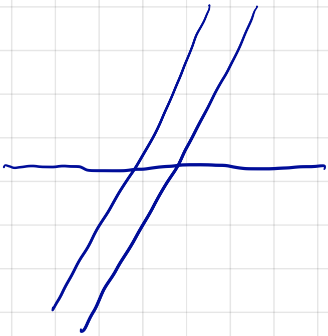
So energy is $g(\underset{\sim}{P}, \underset{\sim}{U})$.

Taking measurements by dot products is common.
Here's another:



particles.

density: $\dfrac{\Sigma}{\Delta x} = \sigma_0$



Chunk has width $\dfrac{1}{\gamma}\Delta x$

and new density $\gamma \dfrac{\Sigma}{\Delta x}$

I.e. New density $= \gamma \sigma_0$

Let me make a vector in the rest frame $\begin{bmatrix} \sigma_0 \\ 0 \end{bmatrix}$

In a frame travelng in which the particles are travelng with velocity v, $\gamma(v) = C$

$$\begin{bmatrix} C & S \\ S & C \end{bmatrix} \begin{bmatrix} \sigma_0 \\ 0 \end{bmatrix} = \begin{bmatrix} C\sigma_0 \\ S\sigma_0 \end{bmatrix}$$

In the time position we have the observed density.

A couple of ways to think about this:

$$\frac{\sigma_0}{c} \underbrace{c \begin{bmatrix} C \\ S \end{bmatrix}}_{\text{4-velocity of the particles.}}$$

Let's multiply by c

$$\underbrace{\sigma_0}_{\text{rest density}} \underbrace{c \begin{bmatrix} C \\ S \end{bmatrix}}_{\text{4-velocity}}$$

components: $c\sigma_0 C \to$ c.o. observed density

$$c\sigma_0 S = c\,\sigma_0 C \left(\frac{S}{C}\right)$$

$$= c\sigma_0 C \,\frac{v}{c}$$

$$= \sigma_0 C\, v$$

components are $\begin{bmatrix} c\sigma \\ \sigma v \end{bmatrix}$ observed density, particle flux

So this is a density-flux vector, N.

If I am observer with 4-velocity V

then in my rest frame

$$g(V, N) = [c, 0]\, G \begin{bmatrix} c\sigma \\ \sigma v \end{bmatrix}$$

$$= c^2 \sigma$$

But this is true in any frame

$$g(\underset{\sim}{V}, \underset{\sim}{N}) = c^2 \sigma$$