What does constant acceleration look like?

Plane $\quad |\alpha''| = \kappa$

$\hookrightarrow |\theta'| = |\alpha''|$ so $\theta' \equiv \kappa$ or $\theta' \equiv -\kappa$

$\theta = \kappa s + s_0$

$$\alpha' = \begin{bmatrix} -\sin(\kappa s + s_0) \\ \cos(\kappa s + s_0) \end{bmatrix}$$

$$\alpha = \frac{1}{\kappa} \begin{bmatrix} \cos(\kappa s + s_0) \\ \sin(\kappa s + s_0) \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$\hookrightarrow$ traverse a circle of radius $\frac{1}{\kappa}$.

Spacetime:

$$\alpha''(\tau) = c \begin{bmatrix} \sinh(\psi(\tau)) \\ \cosh(\psi(\tau)) \end{bmatrix} \psi'(\tau)$$

$$|\alpha''| = \underbrace{c\,|\psi'(\tau)|}_{K}$$

$$\psi(\tau) = \frac{K}{c}\tau + \psi_0$$

$$\alpha''(\tau) = c \begin{bmatrix} \cosh\left(\frac{K}{c}\tau + \psi_0\right) \\ \sinh\left(\frac{K}{c}\tau + \psi_0\right) \end{bmatrix}$$

$$\alpha(\tau) = \frac{c^2}{K} \begin{bmatrix} \sinh\left(\frac{K}{c}\tau + \psi_0\right) \\ \cosh\left(\frac{K}{c}\tau + \psi_0\right) \end{bmatrix} + \begin{bmatrix} ct_0 \\ x_0 \end{bmatrix}$$

$\downarrow$ 0 WLOG

Units of K: $[K c^{-1}] = T^{-1}$

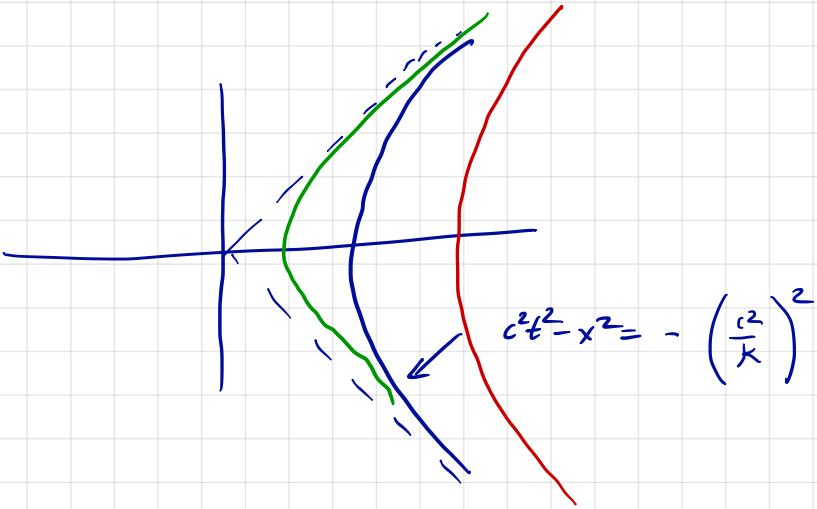
$[K] = T^{-1}[c] = L/T^2$

K is acceleration.

On a curve with $t_0 = 0$, $x_0 = 0$

$$ct^2 - |x|^2 = \left(\frac{c^2}{\kappa}\right)^2 \left[\sinh^2(\cdot) - \cosh^2(\cdot)\right]$$

$$= -\left(\frac{c^2}{\kappa}\right)^2$$



e.g. A rocket accelerates for 10 years (proper!) at 10 m/s.

How much rest time elapses?
How far does it travel w.r.t. the rest-frame.

$$\kappa = 10 \text{ m/s}$$

$$\alpha = \frac{c^2}{\kappa}\left[\begin{array}{l}\sinh\left(\frac{\kappa}{c}\tau\right) \\ \cosh\left(\frac{\kappa}{c}\tau\right)\end{array}\right]$$

1 year $\approx 3\times10^7$ seconds $\quad c \approx 3\times10^8$ m/s =

$\tau = 10$ years $\approx 3\times10^8$ seconds

$\frac{c}{10\text{ m/s}^2} = 3\times10^7\text{ s} = 1$ year

$$\frac{\kappa\tau}{c} = \frac{10}{c}\cdot 3\times10^8 = \frac{3\times10^9}{3\times10^8} = 10$$

rest frame time elapsed $\quad \frac{c}{10}\sinh(10) = 11\,000$ years

distance $\quad c\cdot\frac{c}{10}\left[\cosh(10)-1\right] \approx \frac{c^2}{10}\sinh(10) = 11\,000$ l.y.
11 kly

My center 27 kly, edge 100 kly, andromeda 2 500 kly