(A) and (B) represent the same physical process.

There is a Galilean transformation taking (A) to (B).

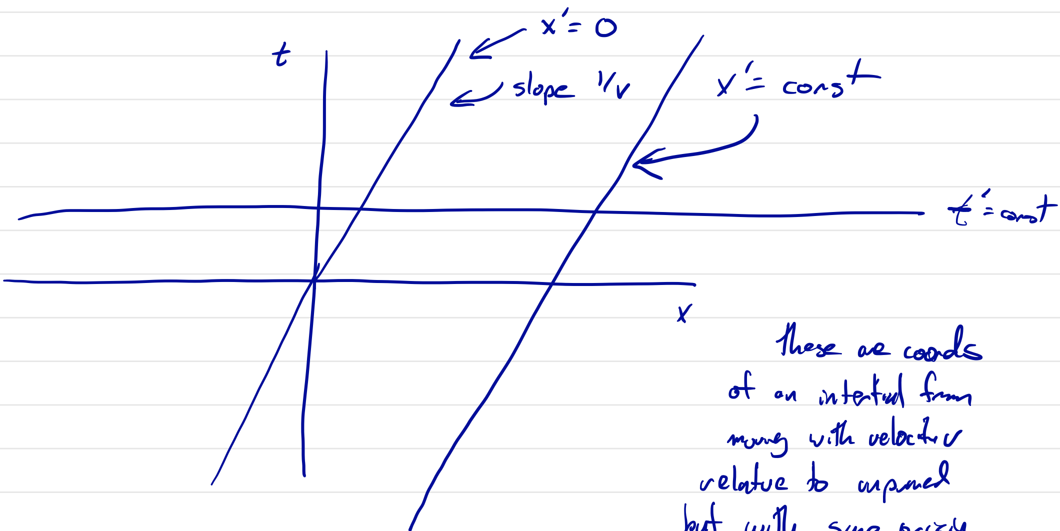
Special category: boosts

$A = I$

$y_0 = 0, \quad t_0 = 0 \qquad \vec{v} = (v, 0, 0)$

$t = t'$

$x = x' + vt' = x' + vt \qquad\qquad t = \frac{1}{v} x$



these are coords of an inertial frame moving with velocity v relative to unprimed but with same origin.

Other special categories

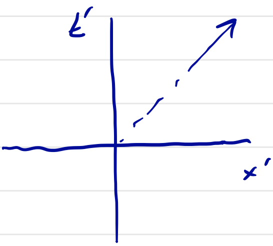
Translations: $(t,x) \longmapsto (t+t_0,\ x+x_0)$

Spatial rotations: $(t,x) \longmapsto (t,\ Hx)$.

Every Galilean transformation is a composition of these three, in the same sense that every isom is a comp of a rotation and a translation.

Alas, the Galilean picture is not the universe we live in.

Observed fact: the speed of light is the same for all observers. Contradicts Galilean relativity



c = speed of light, particle traveling with speed c in pos. x' dir.

$$x'(t') = ct' \qquad t' = t$$

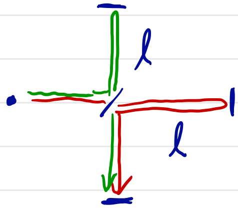
$$\begin{aligned} x(t) &= x'(t) + vt \\ &= ct + vt \\ &= \underbrace{(c+v)}_{\text{not } c!}t \end{aligned}$$

How? Michelson-Morley '89

Premise: Electromagnetic waves are waves in something: aether

Earth is moving with respect to aether.

$\xrightarrow{v}$ (w.r.t. aether)



$c-v \longrightarrow$

$c+v \longleftarrow$

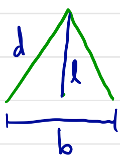
time for red out and back

$$\frac{\ell}{c+v} + \frac{\ell}{c-v} = \frac{2c\ell}{c^2-v^2}$$

$$= \frac{2\ell}{c}\left[\frac{1}{1-\left(\frac{v}{c}\right)^2}\right]$$

$$= \frac{2\ell}{c}\left[1 + \left(\frac{v}{c}\right)^2 + O\left(\frac{v}{c}\right)^4\right]$$

Green: in restframe of aether



Time for one leg: $\frac{d}{c}$

Time for both legs $\frac{2d}{c}$

Distance of b: $\frac{2d}{c} \cdot v = 2d\left(\frac{v}{c}\right)$

$$d^2 = \ell^2 + d^2\left(\frac{v^2}{c^2}\right)$$

$$\frac{d}{dx}\frac{1}{\sqrt{1-x}} = -\frac{1}{2}(1-x)^{-3/2}\cdot(-1)$$

$$= \frac{1}{2}(1-x)^{-3/2}$$

$$d^2\left[1-\left(\frac{v}{c}\right)^2\right] = \ell^2$$

$$d\sqrt{1-\left(\frac{v}{c}\right)^2} = \ell$$

$$\frac{2d}{c} = \frac{2\ell}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

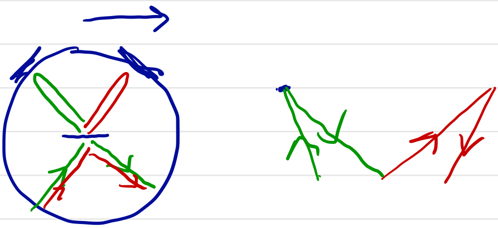
$$= \frac{2\ell}{c}\left[1+\frac{1}{2}\left(\frac{v}{c}\right)^2 + O\left(\left(\frac{v}{c}\right)^4\right)\right]$$

$$= \frac{2\ell}{c} + \frac{\ell}{c}\left(\frac{v}{c}\right)^2 + \frac{2\ell}{c}O\left(\left(\frac{v}{c}\right)^4\right)$$

Time difference: red - green $= \frac{\ell}{c}\left(\frac{v}{c}\right)^2 + \frac{2\ell}{c}O\left(\left(\frac{v}{c}\right)^4\right)$

small compared with

But



By symmetry, path lengths, and time are same.

Waves add constructively:

$$\sin(\omega t) + \sin(\omega t) = 2\sin(\omega t)$$
$$\sin(\omega t + \pi) + \sin(\omega t) = 0$$

leads to interference patterns

So the difference in travel time should cause different inteference patterns for the two configurations, and should smoothly transition as apparatus is rotated.

Instead: pattern is independent of orientation of apparatus.

Conclusion: $v = 0$ w.r.t. aether.

(Full Aether drag)

Inconsistent with two other experiments

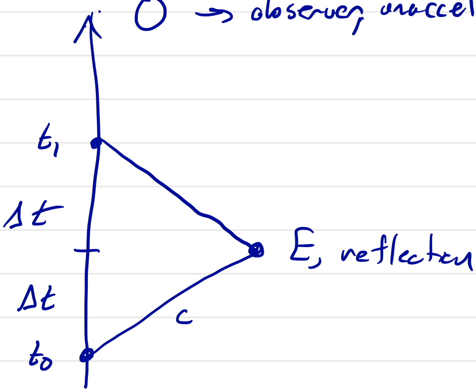
stellar aberration: suggests no aether drag

Fizeau: consistent with partial aether drag

Way out: Light travels with velocity c for all inertial observers. (So in rest frame of experiment, no time difference as apparatus is rotated).

How to put coordinates on spacetime (radar method)

$O \to$ observer, unaccelerating, has a watch.

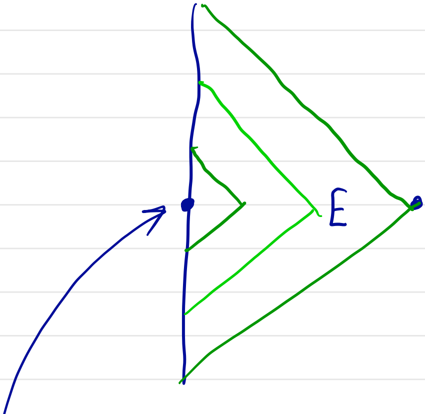


By symmetry E has time coordinate $\dfrac{t_0 + t_1}{2}$.

How far away? $\quad t_1 - t_0 = 2\Delta t$

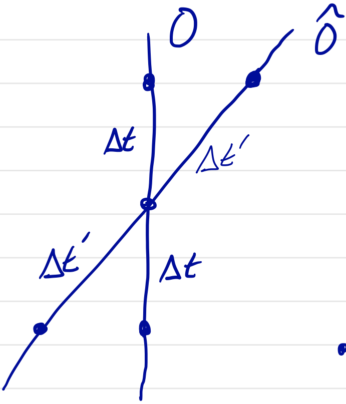
$$\Delta x = c\,\Delta t = \frac{t_1 - t_0}{2}\,c$$

What are events simultaneous with E?

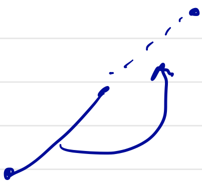


midway time between send/receive is the same.

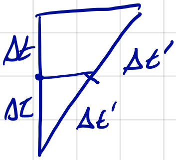
Now consider O' traveling with velocity v relative to O



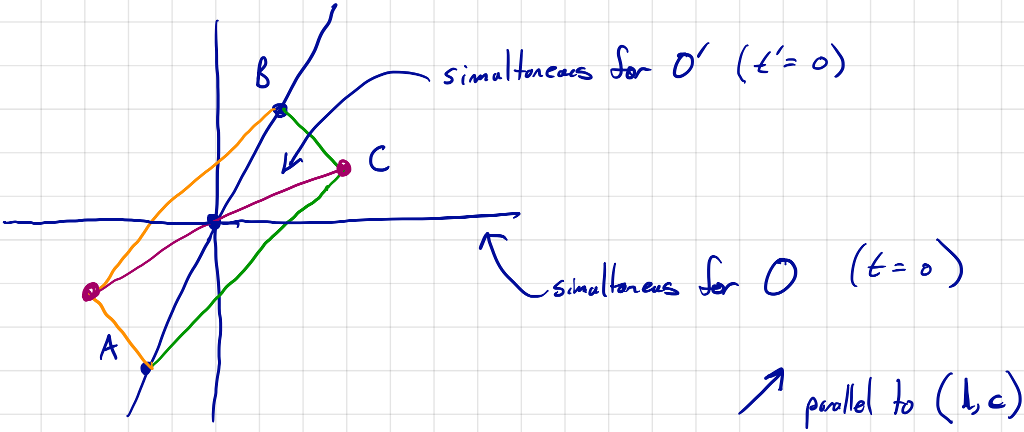
(Translation invariance of time differences)



I



Now suppose O' does radar:



For O: $A = (-\Delta t, -v\Delta t)$
$B = (\Delta t, v\Delta t)$

$$C = A + \lambda_A (1, c)$$

$$= B + \lambda_B (1, -c)$$

OMIT

$$-\Delta t + \lambda_A = \Delta t + \lambda_B \Rightarrow \lambda_A - \lambda_B = 2\Delta t$$

$$-v\Delta t + \lambda_A c = v\Delta t - \lambda_B c = \quad (\lambda_A + \lambda_B)c = 2v\Delta t$$

$$\lambda_A + \lambda_B = 2\left(\frac{v}{c}\right)\Delta t$$

Add: $2\lambda_A = 2\Delta t\left[1 + \left(\frac{v}{c}\right)\right]$

$$\lambda_A = \Delta t\left[1 + \left(\frac{v}{c}\right)\right]$$

$$\lambda_B = \lambda_A - 2\Delta t = \Delta t\left[-1 + \left(\frac{v}{c}\right)\right]$$

$$C = (-\Delta t, -v\Delta t) + \Delta t\left[1 + \left(\frac{v}{c}\right)\right](1, c)$$

$$= \left(\Delta t\left(\frac{v}{c}\right), \Delta t\right)$$

via algebra.

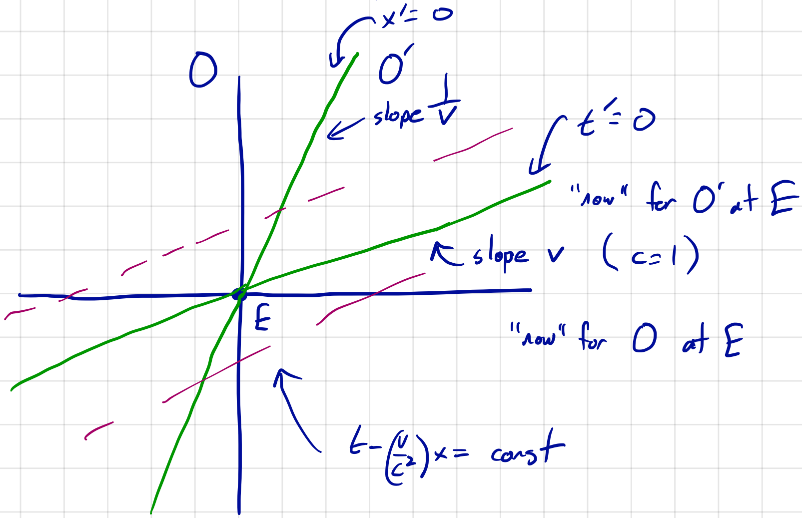
But C is on $t' = 0$.

I.e. C has $t' = 0 \Leftrightarrow C = \Delta t\left[\left(\frac{v}{c}\right), c\right]$ for some Δt.

$C = (t, x)$, on $t' = 0 \Leftrightarrow -ct + \left(\frac{v}{c}\right)x = 0$

i.e. $t = \left(\frac{v}{c^2}\right) x$

It is handy to draw pictures with units $c = 1$.

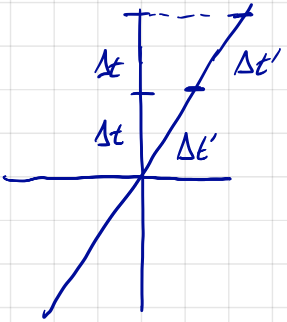


Repeat at different spots on O''s world line and get

$$t - \left(\frac{v}{c^2}\right) x = \text{const} \iff t' = \text{const}$$

$$t' = f\left(t - \left(\frac{v}{c^2}\right) x\right)$$

What's f?



$$\Delta t' = \alpha \Delta t \quad \text{when } x' = 0 \quad (\text{i.e. } x = vt)$$

$$\Downarrow$$

$$t' = \alpha t \quad \text{when } x' = 0$$

$$t' = f\left(t - \frac{v}{c^2} x\right)$$

$$\Downarrow$$

$$\alpha t = f\Big(\underbrace{t\left(1 - \left(\tfrac{v}{c}\right)^2\right)}_{s}\Big) \qquad [x = vt]$$

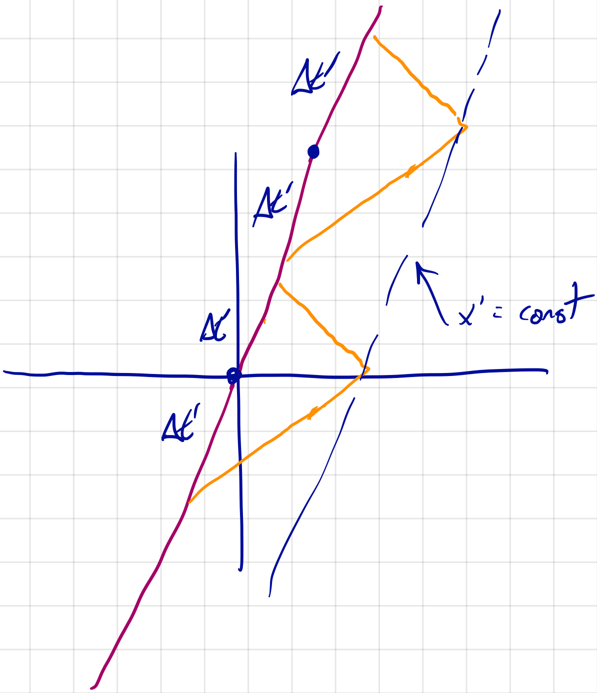
$$\frac{\alpha s}{1 - \left(\frac{v}{c}\right)^2} = f(s)$$

I.e. we've identified f up to α.

$$f(s) = \gamma s \quad \text{for some } \gamma = \frac{\alpha}{1 - \left(\frac{v}{c}\right)^2}$$

$$t' = \gamma \left[t - \frac{v}{c^2} x \right]$$

Similarly, x' is constant on lines parallel to $x = vt$



$$x' = g(x - vt)$$

Value of x' is determined from value when $t' = 0$.

$$\Downarrow$$

$$t = \frac{v}{c^2} x$$

$$x' = g\left(x\left(1 - \left(\frac{v}{c}\right)^2\right)\right)$$

on the line $t' = 0$.

Claim: $x' = \beta x$ on line $t' = 0$ for some β.

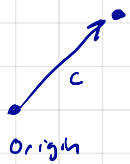
Assuming this, since $t' = 0 \iff t - \frac{v}{c^2} x = 0$

$$\beta x = x' = g(x - vt) = g\left(x\left(1 - \left(\frac{v}{c}\right)^2\right)\right)$$

$$\Rightarrow \quad g(s) = \underbrace{\frac{\beta}{1 - \left(\frac{v}{c}\right)^2}}_{\hat{\gamma} \text{ for now.}} s$$

I.e. $\quad x' = g(x - vt) = \hat{\gamma}(x - vt)$

Moreover: $\hat{\gamma} = \gamma$



(t, ct) O-coords

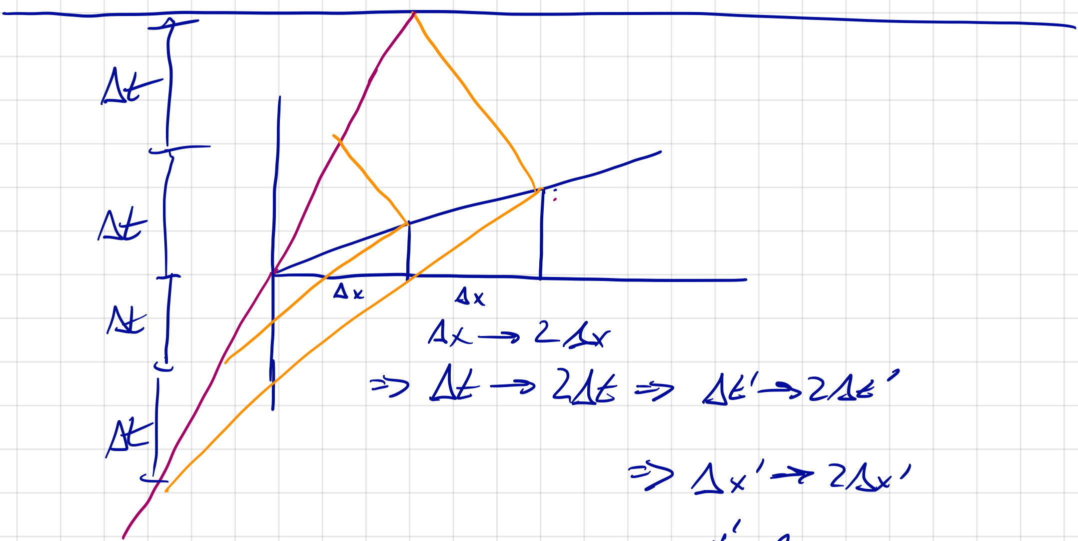
(t', ct') O'-coords

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \gamma t\left(1 - \frac{v}{c}\right)$$

$$x' = \hat{\gamma}(x - vt) = \hat{\gamma}(ct - vt)$$

$$= \hat{\gamma}\, ct\left(1 - \frac{v}{c}\right)$$

$$x' = ct' \Rightarrow \hat{\gamma} = \gamma.$$



$\Delta x \to 2\Delta x$

$\Rightarrow \Delta t \to 2\Delta t \Rightarrow \Delta t' \to 2\Delta t'$

$\Rightarrow \Delta x' \to 2\Delta x'$

$\Rightarrow x' = \beta x$

What's the value of γ?

$$x' = \gamma(x - vt) \qquad x = \gamma'(x' + vt')$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma'(x' + vt')$$

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \gamma \begin{bmatrix} 1 & -\frac{v}{c^2} \\ -v & 1 \end{bmatrix} \begin{pmatrix} t \\ x \end{pmatrix} \qquad \begin{pmatrix} t \\ x \end{pmatrix} = \gamma' \begin{bmatrix} 1 & \frac{v}{c^2} \\ v & 1 \end{bmatrix} \begin{pmatrix} t' \\ x' \end{pmatrix}$$

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma \begin{pmatrix} c & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{v}{c^2} \\ -v & 1 \end{pmatrix} \begin{pmatrix} c^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$= \gamma \begin{pmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \gamma' \begin{pmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix}$$

$$I = \gamma\gamma' \cdot \begin{pmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{pmatrix} = \begin{pmatrix} 1-\left(\frac{v}{c}\right)^2 & 0 \\ 0 & 1-\left(\frac{v}{c}\right) \end{pmatrix}$$

Reasonable symmetry: γ depends on $|v|$ only, so $\gamma = \gamma'$!

$$\gamma^2 = \frac{1}{1-\left(\frac{v}{c}\right)^2} \Rightarrow \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$