## 1. SR 1.1

2. SR 1.2 (i) - (ii)
3. The general linear group $G L(\mathbb{R}, 3)$ is the set of $3 \times 3$ invertible matrics. In this exercise, we show that the Euclidean group $E\left(\mathbb{R}^{2}\right)$ can be seen as a subgroup of $G L(\mathbb{R}, 3)$. (More formally, for the cognoscenti, we construct a group homomorphism from $E\left(\mathbb{R}^{2}\right)$ to $G L(\mathbb{R}, 3)$, i.e. a group representation of $E\left(\mathbb{R}^{2}\right)$.)

If

$$
i(x, y)=\left(\begin{array}{cc}
c & \mp s  \tag{1}\\
s & \pm c
\end{array}\right)\binom{x}{y}+\binom{t_{x}}{t_{y}}
$$

we define

$$
M_{i}=\left(\begin{array}{ccc}
c & \mp s & t_{x}  \tag{2}\\
s & \pm c & t_{y} \\
0 & 0 & 1
\end{array}\right) .
$$

a) Suppose $x, y \in \mathbb{R}$. Show that

$$
M_{i}\left(\begin{array}{l}
x  \tag{3}\\
y \\
1
\end{array}\right)
$$

has the form

$$
\left(\begin{array}{l}
a  \tag{4}\\
b \\
1
\end{array}\right)
$$

and that $i(x, y)=(a, b)$.
b) Show that if $i_{1}$ and $i_{2}$ belong to $E\left(\mathbb{R}^{2}\right)$ then

$$
\begin{equation*}
M_{i_{2} \circ i_{1}}=M_{i_{2}} M_{i_{1}} . \tag{5}
\end{equation*}
$$

Note that on the right-hand side of this equation we are multiplying matrics.
c) Conclude that if $i \in E\left(\mathbb{R}^{2}\right)$, then

$$
\begin{equation*}
M_{i^{-1}}=\left(M_{i}\right)^{-1} . \tag{6}
\end{equation*}
$$

4. [Challenge, not due!] SR 1.2 (iii)
