1. Suppose that $\alpha^a(s)$ is a geodesic. Show that $\alpha^s(ks)$ is for any $k \in \mathbb{R}$.

2. Suppose X^a and Y^a are parallel transported along $\alpha^c(s)$. Show that $g_{ab}X^aY^b$ is constant.

3. GR 5.6

4. Use the result from GR 5.6 to show that the recipe

$$\nabla_a X^b = \frac{\partial X^b}{\partial x^a} + \Gamma^b_{ac} X^c \tag{1}$$

is a tensorial recipe.

5. One way to think about Christoffel symbols is to consider Γ^a_{bc} as a collection of matrices ${M^a}_b$, one for each index c. Show that the following algorithm can be used to compute Γ^a_{bc}.

1. Let u_b be the entries of row c of g_{ab}.
2. Let $A_{ab} = \partial_a u_b$.
3. Let $B_{ab} = A_{ab} - A_{ba}$.
4. Let $C_{ab} = \partial_c g_{ab}$.
5. Let $D_{ab} = (1/2)(C_{ab} - B_{ab})$.
6. Let ${M^a}_b = g^{ac} D_{cb}$.

Then $\Gamma^a_{bc} = {M^a}_b$.