Applied Optimization

(a.k.a. What's this good for anyway?)

We're going to look at word problems where we want to maximize or minimize a desired quantity. (minimize cost, time. Maximize profit, speed)

Two main tools

1) Etreme Value Theorem.

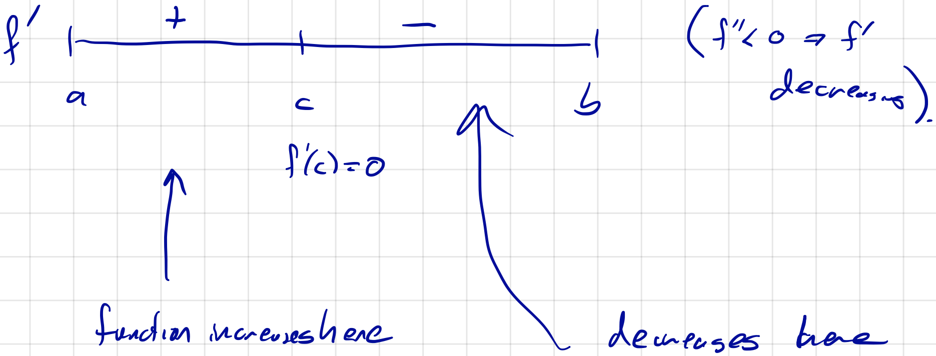
$f(x)$ on $[a,b]$.

↑ closed, bounded.

check crit pts, end points. min/max value is guaranteed to be at one of these few spots.

2) Concavity method.

If f is defined on an interval (a,b), possibly infinite, and $f'(c)=0$ and $f''(x)<0$ on (a,b), then f admits an absolute maximum at c.

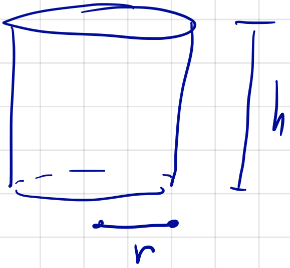


And if $f''(x) > 0$ on (a,b) then f has an abs min at c.

Let's see an example.

Suppose a can has fixed volume V. What dimensions for the can minimize surface area.

1) Read problem!
2) Draw a picture; Label it.



3) Introduce "Q," the quantity to optimize, and write it in terms of other vars

$$A = 2\pi r h + 2\pi r^2$$

4) Use relations to express in terms of just 1 variable.

$$V = \pi r^2 h \Rightarrow h = \frac{V}{\pi r^2}$$

$$A = \frac{2V}{r} + 2\pi r^2$$

5) Now apply calculus.

$$\frac{dA}{dr} = \frac{-2V}{r^2} + 4\pi r \rightarrow \text{crit pts?} \quad 4\pi r = \frac{2V}{r^2}$$

$$r^3 = \frac{V}{2\pi}$$

$$\frac{d^2A}{dr^2} = \frac{4V}{r^3} + 4\pi > 0$$

$$r = \left(\frac{V}{2\pi}\right)^{1/3}$$

for $r > 0$!

So, if there is a crit pt, there is an abs min there!

$$r = \left(\frac{V}{2\pi}\right)^{1/3}$$

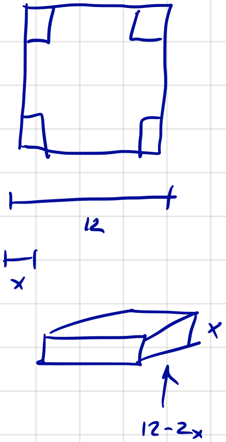
$$h = \frac{V}{\pi r^2} = \frac{V}{\pi}\left(\frac{2\pi}{V}\right)^{2/3} = 2^{2/3}\left(\frac{V}{\pi}\right)^{1/3}$$

$$= 2\left(\frac{V}{2\pi}\right)^{1/3}$$

$$= 2r$$

$h = 2r$!

e.g. We are going to construct an open top box from a 12"×12" square of tin.



What is the maximum possible enclosed volume?

Volume: $V = (12-2x)^2 \cdot x \qquad 0 \le x \le 12$

$$\frac{dV}{dx} = (12-2x)^2 - 2(12-2x)\cdot x \cdot 2$$

$$= (12-2x)\left[12-2x - 4x\right]$$

$$= (12-2x)(12-6x)$$

Closed interval method: check crit pts $(x=6,\ x=2)$

and end pts $(x=0,\ x=6)$

$$V(6) = 0 = V(0)$$

$V(2) = 2 \cdot 8^2 = 128$ in^3. $\longleftarrow$ max volume!