Consider the following problem:

Air is being pumped into a spherical balloon at a rate of 4.5 ft^3/min.

What is the rate of change of the volume of the balloon when its radius is 2 ft

We have two related quantities: V: volume of balloon
r: radius of balloon

$$V = \frac{4}{3}\pi r^3$$

One of the quantities is changing: V

So the other (r) must as well.

We know the rate of change of V $\left(\frac{dV}{dt} = 4.5\ ft^3/min\right)$

Can we deduce the rate of change of r?

$$V(t) = \frac{4}{3}\pi\left(r(t)\right)^3$$

Take a derivative with respect to t:

$$V'(t) = \frac{4}{3}\pi\, 3\left(r(t)\right)^2 r'(t)$$

(just the chain rule)

You'll drop the (t)'s, though:

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\, 3r^2 \frac{dr}{dt}$$

in effect, this is implicit diff w.r.t. t.

$$4.5 = \frac{4}{3}\pi \cdot 3 \cdot 2^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{4.5}{16\pi} = 0.09 \text{ ft/min}$$

This class of problem is known as a related rate problem.

We have two quantities that are related to each other (V, r)

We know how one is changing and we want to know how the other is changing.

1) Identify the quantity you know a rate of change of $(V : dV/dt)$

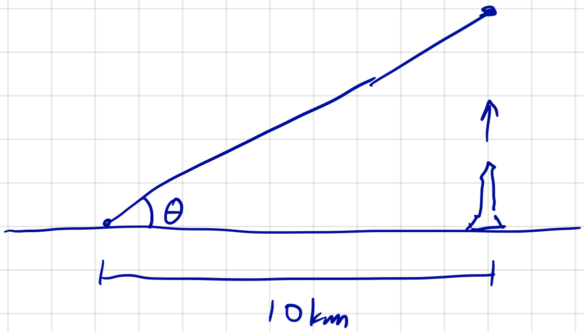
2) Identify the quantity you want a rate of change of $(r : dr/dt)$

3) Find an equation relating the two quantities $\left(V = \frac{4}{3}\pi r^3\right)$

4) Take an implicit derivative of both sides of the equation $\left(\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}\right)$

5) Substitute for all known data and solve for the rate of change you want.

e.g.



A camera 10 km from a launch site is tracking a rocket that is rising vertically.

How fast is the rocket rising if the camera angle θ is increasing at a rate of 0.5 rad / min when the angle is $\pi/3$ radians?

Know : $\theta = \pi/3$ rad

$\frac{d\theta}{dt} = 0.5$ rad / min

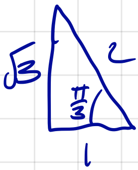
Want $\frac{dh}{dt}$

$\rightarrow$ solution

$$\boxed{\tan\theta = \frac{h}{10}}$$

$$\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{10}\frac{dh}{dt}$$

$$\frac{dh}{dt} = \sec^2\theta \; 10\frac{d\theta}{dt}$$



$$\cos(\pi/3) = \frac{1}{2}$$

$$\sec(\pi/3) = 2$$

$$\frac{dh}{dt} = 4 \cdot 10(km) \cdot \frac{1}{2}$$

$$= 20 \text{ km/min}$$