Finish up:

$$h'(t) = -3e^{-3/2 t}\sin(2\pi t) + 4\pi e^{-3/2 t}\cos(2\pi t)$$

$$= e^{-3/2 t}\left[-3\sin(2\pi t) + 4\pi\cos(2\pi t)\right]$$

$$h'(1) = e^{-3/2} \cdot 4\pi = 2.80\ldots \quad \text{cm/s}$$

$$\lim_{t\to\infty} h(t) = 0$$ ~~goes~~ comes to a rest.

$$y = 2\cos(x) + \cos^2(x)$$

$$y' = -2\sin(x) + 2\cos(x)\sin(x)$$

$$= 2\sin(x)\left[-1 + \cos(x)\right]$$

$y' = 0$: $\quad x = k\pi \quad k \in \mathbb{Z}$ $\qquad x = k\pi$

$x = \pi + l2\pi \quad l \in \mathbb{Z}$ $\qquad x = (2l+1)\pi$

$x = k\pi$

$$y' = -\sin(\sin(3x)) \cdot \frac{d}{dx}\sin(3x)$$

$$= -\sin(\sin(3x))\, 3\cos(3x)$$

$$= -3\cos(3x)\sin(\sin(3x))$$

$$\frac{d}{dx}\sin(\sin(3x)) = \cos(\sin(3x)) \cdot 3\cos(3x)$$

$$= 3\cos(3x)\cos(\sin(3x))$$

$$y'' = -3(-\sin(3x) \cdot 3)\sin(\sin(3x))$$

$$-3\cos(3x)\left[3\cos(3x)\cos(\sin(3x))\right]$$

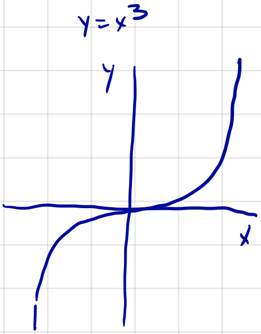
$$= 9\sin(3x)\sin(\sin(3x))$$

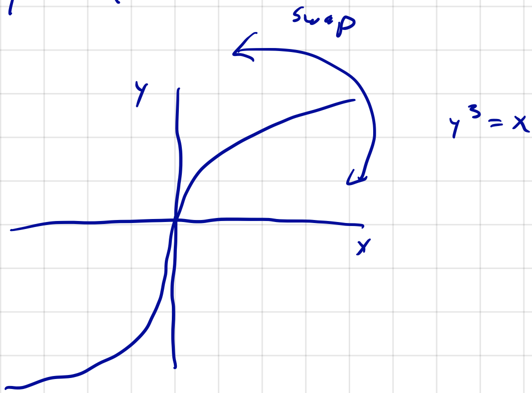
$$-9\cos^2(3x)\left[\cos(\sin(3x))\right]$$

Consider $f(x) = x^{1/3}$

Graph is the curve $y = x^{1/3}$

$y^3 = x$





How can I compute $f'(x)$?
Alt: how can I find $\frac{dy}{dx}$ if $y = x^{1/3}$?

Technique is called implicit differentiation.

$$y^3 = x$$

$$\frac{d}{dx} y^3 = \frac{d}{dx} x$$

$$3y^2 \frac{dy}{dx} = 1$$

This is the chain rule. $\left(\text{If you want, } \frac{dy^3}{dy} \cdot \frac{dy}{dx}\right)$

$$\frac{dy}{dx} = \frac{1}{3y^2}$$

The y is unfortunate.

I want a function of x!

But $y^3 = x$

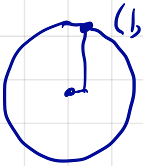
$$y = x^{1/3}$$
$$y^2 = x^{2/3}$$

$$\frac{dy}{dx} = \frac{1}{3x^{2/3}} = \frac{1}{3} x^{-2/3}$$

Silly geometric fun:

The point $(1, 4)$ lies on the curve $x^2+y^2=17$.

Find the slope of the tangent line at this point.

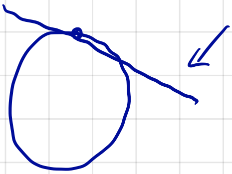


$$\frac{d}{dx}\left(x^2+y^2\right) = \frac{d}{dx}17$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}.$$

At $(1, 4)$ the slope is then $-\frac{1}{4}$



$$(y-4) = -\frac{1}{4}(x-1)$$

$$y = 4 - \frac{1}{4}(x-1)$$

e.g. $xe^y = x - y$ $\qquad x=-1, \; y=0$

$$\frac{d}{dx}\left(xe^y\right) = \frac{d}{dx}\left(x-y\right)$$

$$e^y + xe^y\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$xe^y\frac{dy}{dx} + \frac{dy}{dx} = 1 - e^y$$

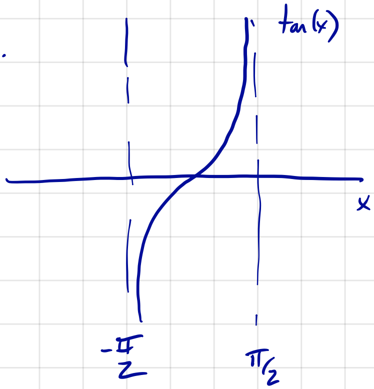
$$\frac{dy}{dx} = \frac{1-e^y}{1+xe^y} \leftarrow \frac{1-1}{1-1} \text{ at}$$

One point on the line is $(x,y)=(0,0)$

At that point, $\frac{dy}{dx} = \frac{1-e^0}{1+0e^0} = \frac{0}{1} = 0$

Another is $(1,0)$ $\qquad \frac{1-e^0}{1+1\cdot 1} = \frac{0}{2} = 0$

e.g.



arctan(x) is an angle y with $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and

$\tan(y) = x$

$\tan(\arctan(x)) = x$

$\arctan(\tan(y)) = y$ so long as $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$y = \arctan(x)$

$\tan(y) = x$

$\frac{d}{dx}\tan(y) = \frac{d}{dx}x$

$\sec^2(y)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2(y)}$

Rather have x. But

$\sec^2(y) - \tan^2(y) = 1$

$\sec^2(y) = 1 + \tan^2(y) = 1 + x^2$

So $\frac{dy}{dx} = \frac{1}{1+x^2}$.

That is $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$. See text for remaining ones.

Now you try:

Find $\frac{dy}{dx}$ if $y\sin(x) = x^2 - y^2$

Find $\frac{d^2y}{dx^2}$ if $x^2 + 2y^2 = 2$