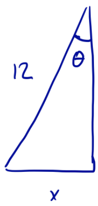
2. If you increase the radius of a snowball from 2 inches to 2.02 inches, estimate the change in volume of the snowball.

$$\Delta r = 2.02 - 2 = 0.02.$$

$$\Delta V \approx V'(2) \cdot \Delta r = 50.265 \cdot (0.02) = 1.0053 \text{ cubic inches}$$

6. A 12 foot ladder rests against a wall. Let θ be the angle between the ladder and the wall and let x be the distance from the base of the ladder and the wall.

a. Compute x as a function of θ.

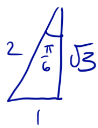


$$\sin\theta = \frac{x}{12}$$

$$x = 12\sin\theta$$

b. How fast does x change with respect to θ when $\theta = \pi/6$?

$$x(\theta) = 12\sin\theta$$



$$x'(\theta) = 12\cos\theta$$

$$x'\left(\frac{\pi}{6}\right) = 12\cos\frac{\pi}{6}$$

$$= 12 \cdot \frac{1}{2} = 6.$$

Chain Rule

We've seen: $\frac{d}{dx} e^{2x} = 2e^{x}$

$$\frac{d}{dx} e^{-x} = - e^{-x}.$$

There is a pattern here. To see more, consider

$$\frac{d}{dx} \sin(5x) = \lim_{h \to 0} \frac{\sin(5(x+h)) - \sin(5x)}{h}$$

$$= 5 \lim_{h \to 0} \frac{\sin(5x + 5h) - \sin(5x)}{5h}$$

$$= 5 \lim_{w \to 0} \frac{\sin(5x + w) - \sin(5x)}{w}$$

$$= 5 \sin'(5x)$$

$$= 5 \cos(5x)$$

In general $\frac{d}{dx} \sin(ax) = a \cos(ax)$

$$\frac{d}{dx} e^{ax} = a e^{ax}$$

These are special cases of the following rule, known as the Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

$$\frac{d}{dx} \sin(5x) = \sin'(5x) \cdot \frac{d}{dx}(5x)$$

$$= 5\cos(5x)$$

e.g. $$\frac{d}{dx} \cos(x^2) = -\sin(x^2) \frac{d}{dx} x^2$$

$$= 2x \sin(x^2)$$

Now your turn, Worksheet - 3.4.