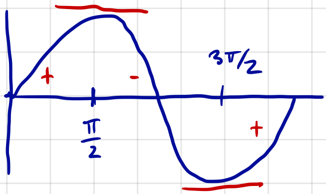
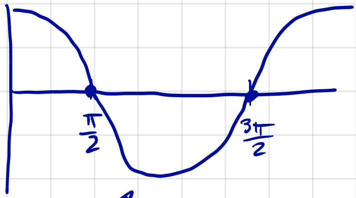
Two more derivatives:





looks kinda like cos(x).

But kinda isn't exact.

let's show $\frac{d}{dx}\sin(x) = \cos(x)$ exactly.

Keys: 1) $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$ $\qquad \frac{0}{0}$

2) $\lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0$ $\qquad \frac{0}{0}$

Assuming these for the moment, we'll also need

3) $\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$ ← trig identity.

4) $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

With this:

$$\frac{d}{dx}\sin(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

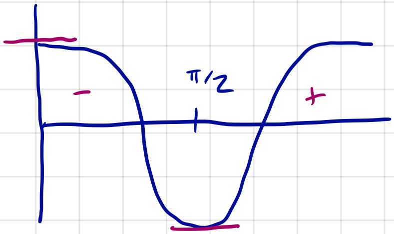
$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \left[\sin(x)\frac{[\cos(h) - 1]}{h} + \cos(x)\frac{\sin(h)}{h} \right]$$

$$= \sin(x) \lim_{h\to 0} \frac{[\cos(h) - 1]}{h} + \cos(x) \lim_{h\to 0} \frac{\sin(h)}{h}$$

$$= \sin(x)\cdot 0 + \cos(x)\cdot 1$$

$$= \cos(x).$$



$$\lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h} = \lim_{h \to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$

$$= \lim_{h \to 0} \cos(x) \frac{[\cos(h) - 1]}{h} - \lim_{h \to 0} \sin(x) \frac{\sin(h)}{h}$$

$$= \cos(x) \cdot 0 - \sin(x) \cdot 1$$

$$= -\sin(x)$$

So it boils down to two limits: a) $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

b) $\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0.$

In fact, if I can get a) I'll also get b).:

$$\sin^2(x) + \cos^2(x) = 1$$

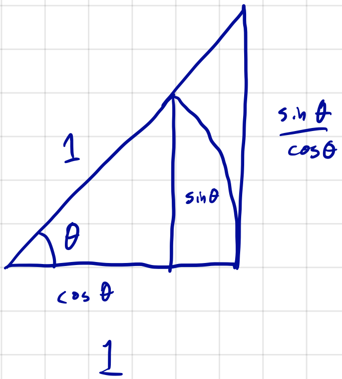
$$\sin^2(x) = 1 - \cos^2(x)$$
$$= (1 - \cos(x))(1 + \cos(x))$$

$$1 - \cos(x) = \frac{\sin^2(x)}{1 + \cos(x)}$$

$$\frac{1 - \cos(x)}{x} = \frac{\sin(x)}{x} \quad \frac{\sin(x)}{1 + \cos(x)} \qquad (x \neq 0)$$

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x} = 1 \cdot \frac{\sin(0)}{1 + 1} = 0$$

So now it boils down to $\lim_{x \to 0} \frac{\sin(x)}{x} = 0.$



area: $\frac{\theta}{2\pi} \cdot \pi = \frac{\theta}{2}$

$$\frac{1}{2} \cos\theta \sin\theta \leq \frac{\theta}{2} \leq \frac{1}{2} \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq \frac{1}{\cos\theta}$$

As $\theta \to 0$, $\cos\theta \to 1$, $\frac{1}{\cos\theta} \to 1$, so $\frac{\sin\theta}{\theta} \to 1$.

Other trig derivatives:

$$\frac{d}{dx}\tan(x) = \frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \sec^2(x)$$ (You did this!)

$$\frac{d}{dx}\cot(x) = \frac{d}{dx}\frac{\cos(x)}{\sin(x)} = \frac{-\sin^2 x - \cos^2(x)}{\sin^2(x)} = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \frac{d}{dx}\frac{1}{\cos(x)} = -\frac{-\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)}\frac{1}{\cos(x)} = \sec x \tan x$$

$$\frac{d}{dx}\sec(x) = \frac{d}{dx}\frac{1}{\sin(x)} = \frac{-\cos(x)}{\sin^2(x)} = -\cot x \csc(x)$$

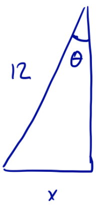
2. If you increase the radius of a snowball from 2 inches to 2.02 inches, estimate the change in volume of the snowball.

$$\Delta r = 2.02 - 2 = 0.02.$$

$$\Delta V \approx V'(2)\cdot \Delta r = 50.265\cdot(0.02) = 1.0053 \text{ cubic inches}$$

6. A 12 foot ladder rests against a wall. Let θ be the angle between the ladder and the wall and let x be the distance from the base of the ladder and the wall.

a. Compute x as a function of θ.

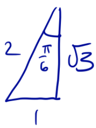


$$\sin\theta = \frac{x}{12}$$

$$x = 12\sin\theta$$

b. How fast does x change with respect to θ when $\theta = \pi/6$?

$$x(\theta) = 12\sin\theta$$



$$x'(\theta) = 12\cos\theta$$

$$x'\left(\frac{\pi}{6}\right) = 12\cos\frac{\pi}{6}$$

$$= 12\cdot\frac{1}{2} = 6.$$