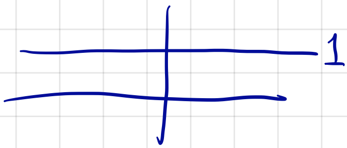
I'm tired of computing derivatives the hard way!

We need some rules.

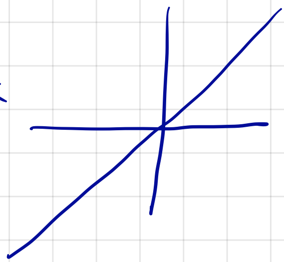
$f(x) = 1$



$f'(x) = 0$

Alternative notation: $\frac{d}{dx} 1 = 0$

$f(x) = x$



$f'(x) = 1$

$\frac{d}{dx} x = 1$

e.g. $f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2}{h}$$

$$= \lim_{h \to 0} 2x + h = 2x$$

On worksheet: $\frac{d}{dx} x^3 = 3x^2$

Rules for combining:

$$\frac{d}{dx} 7x^2 = 7 \frac{d}{dx} x^2 = 7 \cdot (2x) = 14x.$$

constant multiple rule,

comes from constant multiple rule for limits.

$$\lim_{h\to 0} \frac{7(x+h)^2 - 7x^2}{h} = \lim_{h\to 0} 7\,\frac{\left((x+h)^2 - x^2\right)}{h}$$

$$= 7 \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = 7 \cdot 2x$$

In general $\frac{d}{dx}\, c f(x) = c \frac{d}{dx} f(x)$.

In fact: $\frac{d}{dx} x^n = n x^{n-1}$ $\quad n = 1, 2, 3, \ldots\ldots$ (Next class)

In fact $\frac{d}{dx} \cdot x^a = a x^{a-1}$ $\quad$ if $x > 0$, $a \in \mathbb{R}$ (Much later)

How about $f(x) = 2x^2 - 5x + 9$?

$$\frac{d}{dx}\left[2x^2 - 5x + 9\right] = \frac{d}{dx}\left[2x^2\right] + \frac{d}{dx}\left[-5x\right] + \frac{d}{dx}\left[9\right]$$

$$= 2\frac{d}{dx}\left[x^2\right] - 5\frac{d}{dx}\left[x\right] + 9\frac{d}{dx}\left[1\right]$$

$$= 2\left[2x\right] - 5\cdot 1 + 9\cdot 0$$

$$= 4x - 5$$

In general $\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$

Comes from the sum rule for limits.

One last function:

$$f(x) = 2^x$$

$$f'(x) = \lim_{h\to 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h\to 0} 2^x \frac{(2^h - 1)}{h}$$

$$= 2^x \lim_{h\to 0} \frac{2^h - 1}{h}$$

$$= 2^x \cdot \underbrace{f'(0)}_{\approx 0.693}$$

$$f(x) = 10^x$$

$$f'(x) = 10^x \underbrace{f'(0)}_{\approx 2.302}$$

There is a number e $\quad 2 < e < 10$ $\quad$ where $f(x) = e^x$ has

$f'(0) = 1$ and have

$$\frac{d}{dx} e^x = e^x.$$

(We'll get 2^x, 10^x later...)