Review

Topics I care about:

- curve sketching without a calculator
  (piecewise curves!)

- exponents / logarithms.

Know rules
Know how to solve:

half lives, doubling

$3^x = 5$

(population doubles every 30 minutes:

$\log_{10} 3^x = \log_{10} 5$

$$p = C 2^{t/30})$$

$x \log_{10} 3 = \log_{10} 5$

$$x = \frac{\log_{10} 5}{\log_{10} 3}$$

$$p = C\, 10^{at}$$

$$\frac{t}{30} = \log_{10} 2$$

$$at = 1 \qquad \frac{1}{a} = 30 \log_{10} 2$$

- average rates of change ($[a,b]$
  $[a,a+h]$ versions)

Units!

- instantaneous rates of change via limits

$$f'(a) = \lim_{b\to a} \frac{f(b)-f(a)}{b-a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

You see it via the slopes of secant / tangent lines

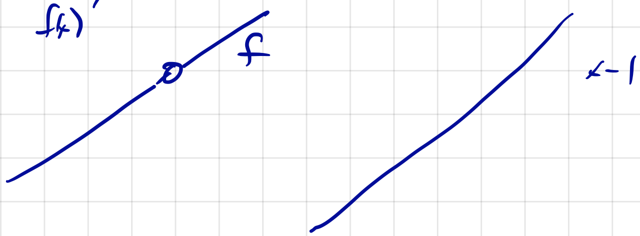
Flavors of limits:

$\frac{0}{0}$, $\pm\infty$, limit at ∞.

↳ we care most about these.

From quiz: $\frac{(x-1)(x-2)}{x-2} \neq x-1$ why?

$f(x)$



$\frac{0}{0}$ examples

$$\frac{x^2-25}{x+25}$$

$$\frac{\sqrt{x}-4}{x-2}$$

$$\frac{\sin(x)}{x}$$

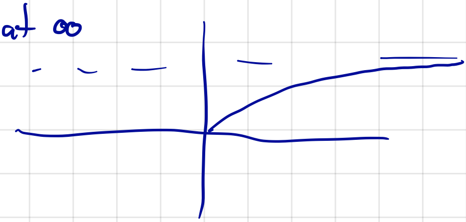
These are important because of the transition from average rates of change to instantaneous.

$\pm\infty$ These encode vertical asymptotes. $\frac{5}{0}$

Important part is the sign:

$$\frac{5}{0^-} \text{ vs } \frac{5}{0^+}$$

limits at ∞



related to horizontal asymptotes.

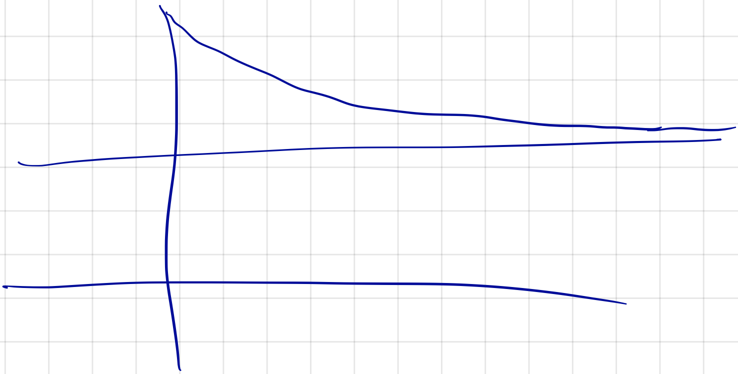
$$\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$$

for large values of x $\arctan(x) \approx \pi/2$.

E.g: $T(t) = 68 + 25e^{-t/5}$

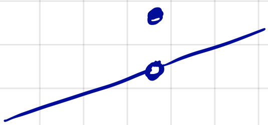
$$\lim_{t\to\infty} T(t) = 68$$

What does this mean?



left / right limits: why? limits disagree $\Rightarrow$ no limit.

infinite limits often need this



limit exists?

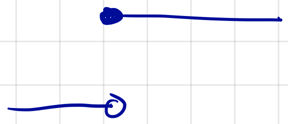
Continuity:

cts at a if

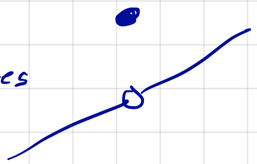
1) a is in domain.
2) $\lim_{x\to a} f(x)$ exists
3) $f(a) = \lim_{x\to a} f(x)$

What goes wrong: usually $\lim_{x\to a} f(x)$ DNE

e.g. $$\boxed{\begin{matrix}\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0.\end{matrix}}$$

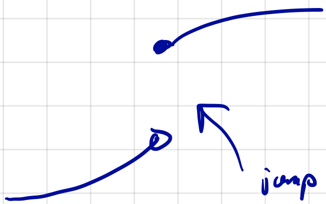
e.g. 

less common: limit exists, but value disagrees



$$\lim_{x\to a} f(x) = f(a)$$

continuity is a key property in limit computations



jump discont.

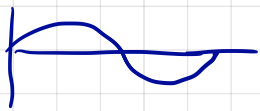
$$\lim_{x\to a^+} f(x) = L$$

$$\lim_{x\to a^-} f(x) = M \qquad L \neq M.$$

$$\frac{0}{0} \to ? \qquad \frac{\sin(x)}{x} \to 1$$

$$\frac{5}{0^+} \to +\infty \qquad \lim_{x\to\pi^+} \frac{5}{\sin(x)} = \frac{5}{0^-} = -\infty.$$

use this!



Limits at ∞:

convert $\frac{\infty}{\infty}$ into something else

(see worksheet).

IVT: $f(a) \leq N \leq f(b)$

$[a,b]$ in domain

$\boxed{f \text{ continuous}}$

$\Rightarrow \exists\ x,\ f(x) = N.$

Roots!

Two choices:

$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a} = \lim_{b \to a} \frac{f(a) - f(b)}{a - b}$$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Graphing

Tangent line: $\overset{y_0}{(x_0, f(x_0))}$

slope $f'(x_0)$

$$y - y_0 = m(x - x_0)$$

$f(x_0)$ $\quad$ $f'(x_0)$ $\quad$ x_0 all numbers.

Approximation

$$\frac{\Delta f}{\Delta x} \approx f'(a)$$

$$\frac{f(b) - f(a)}{b - a} \approx f'(a) \quad \text{for } b \text{ near } a.$$

$$f(b) \approx f(a) + f'(a)(b - a)$$