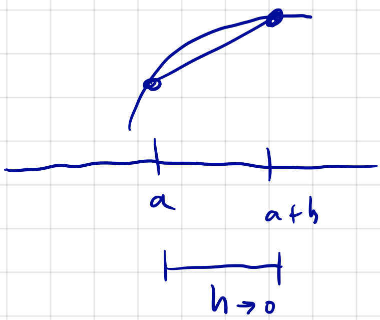
last class:

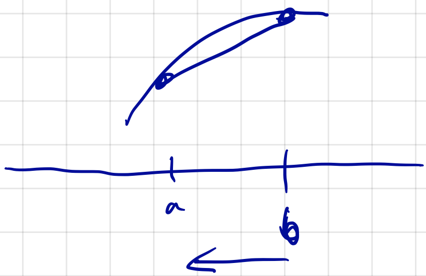
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{b \to a} \frac{f(b) - f(a)}{b - a}$$

1) derivative of f at a

2) instantaneous rate of change of f at a

3) slope of tangent line of graph of f at a.





How to think about the derivative.

Population $P(t)$

$$P'(a) = \lim_{b \to a} \frac{P(b) - P(a)}{b-a} \qquad \frac{\Delta P}{\Delta t} \qquad \Delta t \text{ small}$$

$$P'(a) \approx \frac{\Delta P}{\Delta t} \qquad \Delta t \text{ small}$$

$$\Delta P \approx P'(a)\,\Delta t \qquad \Delta t \text{ small}$$

If time changes by a little Δt from $t = a$

the population will change by ΔP.

$\Delta P = P(b) - P(a)$ so

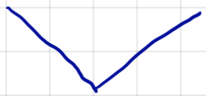
$$P(b) \approx \boxed{P(a) + P'(a)\,\Delta t}$$

only involves stuff at a.

(You'll do this on worksheet)

Alas, not every function has a derivative at every point.

e.g. $f(x) = |x|$

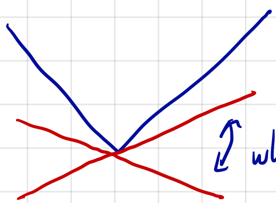


$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h)}{h} = \lim_{h \to 0} \frac{|h|}{h}.$$

For $h > 0$, $|h| = h$ so $\displaystyle\lim_{h \to 0^+} \frac{|h|}{h} = \lim_{h \to 0^+} \frac{h}{h} = 1$

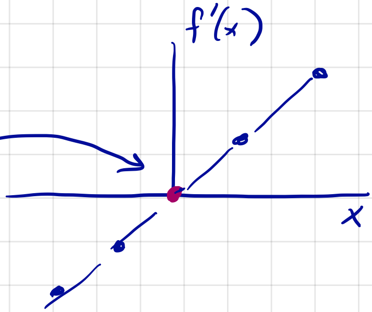
and $\displaystyle\lim_{h \to 0^-} \frac{|h|}{h} = \frac{-h}{h} = -1.$

$\neq$



who gets to be tangent line?

A perspective on the derivative



$f'(0) = 0$

The function $f'(x)$ is called the derivative of f.

eg: $r(t) = 2\sqrt{t}$

r'

$$f'(2) = \lim_{b \to 2} \frac{b^2 - 4}{b - 2}$$

$$= \lim_{b \to 2} \frac{(b-2)(b+2)}{b-2}$$

$$= \lim_{b \to 2} b+2 = 4$$

$$f'(a) = \lim_{b \to a} \frac{b^2 - a^2}{b - a}$$

$$= \lim_{b \to a} \frac{(b-a)(b+a)}{b-a}$$

$$= \lim_{b \to a} b+a$$

$$= a + a = 2a$$

Or use x instead of a:

$$f'(x) = \lim_{b \to x} \frac{b^2 - x^2}{b - x}$$

$$= \lim_{b \to x} \frac{(b-x)(b+x)}{b-x}$$

$$= \lim_{b \to x} b+x = x+x = 2x.$$

$$\boxed{f'(x) = 2x}$$