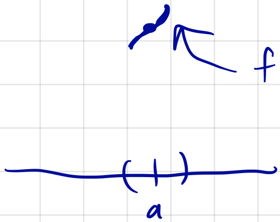
Last class: continuity at a

$$\lim_{x \to a} f(x) = f(a)$$

algebraic: "direct substitution"

graphical



$f(x)$ near a.

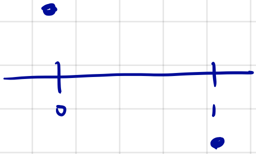
f is continuous if cts at every point.

Important theorem:

Consider $x^5 - 3x + 1 = p(x)$

$p(0) = 1$

$p(1) = -1$

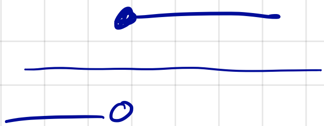


Somewhere in $[0,1]$ is a spot x where $p(x) = 0$.

This doesn't work for discontinuous functions

e.g.

$$f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$



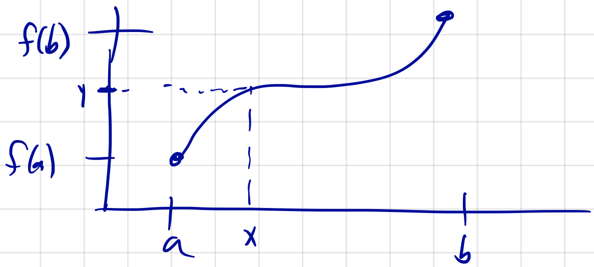
$f(x) \neq 0$ ever!

Intermediate Value Theorem

If $f(x)$ is a continuous function defined on an interval $[a,b]$, for any y between $f(a)$ and $f(b)$

there is $x \in [a,b]$ with $f(x) = y$.

[In particular, if $f(a) \geq 0$ and $f(b) \leq 0$ there is x in $[a,b]$ with $f(x) = 0$.]



(!)

Caveats 1) continuity is necessary

2) domain is an interval

e.g. $f(x) = \dfrac{1}{x}$

$f(1) = 1$

$f(-1) = -1$

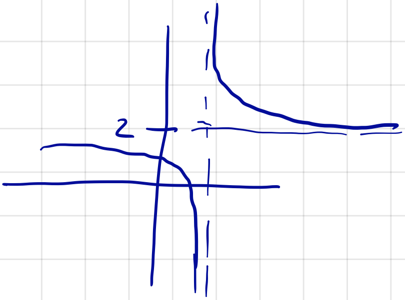
is $f(x) = 0$ ever?

In groups

a) show there is a number x with $10^x = x^2$

Limits at ∞.

$$\frac{1}{x-1} + 2 = \frac{2x-1}{x-1}$$



When x is large, $\frac{2x-1}{x-1} \approx \frac{2x}{x} = 2$.

($2\cdot 1$billion$-1 \approx 2\cdot 1$billion!)

We'll express this via $\lim_{x\to\infty} \frac{2x-1}{x-1} = 2$.

Here's how we can justify:

top $\to \infty$
bottom $\to \infty$

$\frac{\infty}{\infty} \to$ indeterminate

$\frac{0}{0} \to$ also

Instead:

$$\lim_{x\to\infty} \frac{2x-1}{x-1} = \lim_{x\to\infty} \frac{2-\frac{1}{x}}{1-1/x}$$

(multiply top + bottom by $\frac{1}{x}$)

Now: $\lim_{x \to \infty} \frac{1}{x} = 0$.

So $\lim_{x\to\infty} \frac{2 - \frac{1}{x}}{1 - \frac{1}{x}} = \frac{\lim_{x\to\infty} 2 - \frac{1}{x}}{\lim_{x\to\infty} 1 - \frac{1}{x}} = \frac{2}{1}$

Facts you will need:

① $\lim_{x\to\infty} \frac{1}{x} = 0$

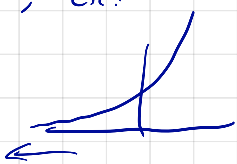
$\lim_{x\to\infty} \frac{1}{x^2} = 0$ $\qquad \lim_{x\to\infty} \frac{1}{x^n} = 0 \quad n > 0.$

$\lim_{x\to\infty} \frac{1}{5x} = 0$

Also, $\lim_{x\to -\infty} \frac{1}{x} = 0$

$\frac{1}{x^2} = 0$, etc.

② $\lim_{x\to -\infty} 10^x = 0$



3:

$$\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$$

$$\lim_{x\to -\infty} \arctan(x) = -\frac{\pi}{2}$$

{Read text about relation between asymptotes + infinite limits}.