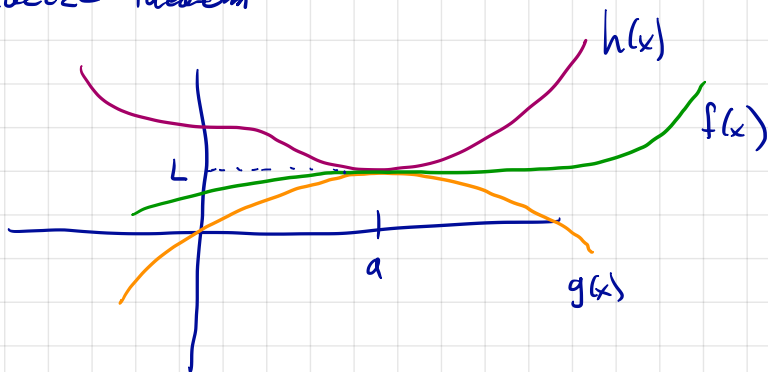


### ③ Squeeze theorem



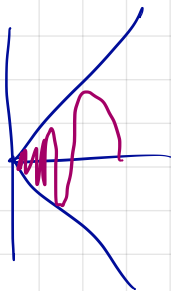
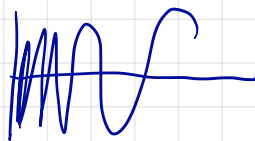
$$\text{if } g(x) \leq f(x) \leq h(x)$$

$$\text{and } \lim_{x \rightarrow a} g(x) = L = \lim_{x \rightarrow a} h(x) \text{ then}$$

$$\lim_{x \rightarrow a} f(x) = L \text{ also.}$$

(Has 1-sided versions, too)

e.g.  $\sin(1/x)$



$$\lim_{x \rightarrow 0^+} x \sin(1/x) :$$

$$\begin{aligned} -1 &\leq \sin(1/x) \leq 1 \\ -x &\leq x \sin(1/x) \leq x \end{aligned}$$

$$\lim_{x \rightarrow 0^+} -x = 0, \quad \lim_{x \rightarrow 0^+} x = 0 \Rightarrow \lim_{x \rightarrow 0^+} x \sin(1/x) = 0 \text{ also.}$$