Big ideas:

average rate of change $\qquad \dfrac{\text{change in dist}}{\text{change in time}} \qquad \dfrac{\Delta d}{\Delta t} \qquad \dfrac{d(t_1)-d(t_0)}{t_1-t_0}$

But if $t_1 = t_0$: $\dfrac{0}{0}$

$P(t) = 1000\,(1.1)^t$

What is rate of change right at $t = 1$?

$$\frac{P(2)-P(1)}{2-1} = 110$$

$$\frac{P(1.1)-P(1)}{1.1-1} = 105.34$$

$$\frac{P(1.01)-P(1)}{1.01-1} = 104.89$$

$$\frac{P(1.001)-P(1)}{1.001-1} = 104.8416$$

$h = 0.1 \qquad 105.34\ldots$

$h = 0.01 \qquad 104.89\ldots$

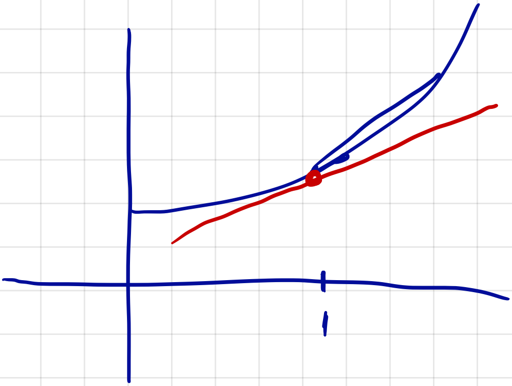
$h = 0.0001 \qquad 104.84169$

$h = 0.0000001 \qquad \underbrace{104.841}\,199$

We can't take $t_1 = 1$. But

$$\lim_{t_1 \to 1} \frac{p(t_1) - p(1)}{t_1 - 1} = 104.8\ldots.$$

This is the instantaneous rate of change of pop at $t=1$.

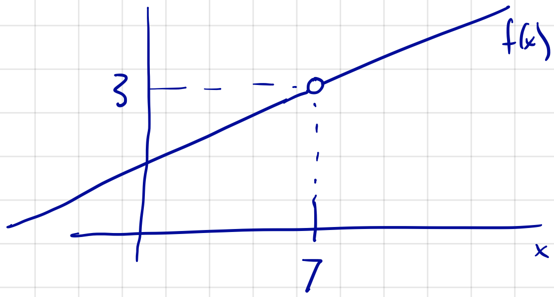


slope of secant lines correspond to average rates of change.

slope of tangent line ~~is equal to~~ corresponds to an instantaneous rate of change.

Last class:

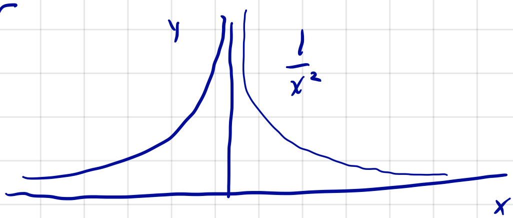
Picture:



$$\lim_{x \to 7} f(x) = 3$$

There are some variations on the limit theme you need to know about:

Consider

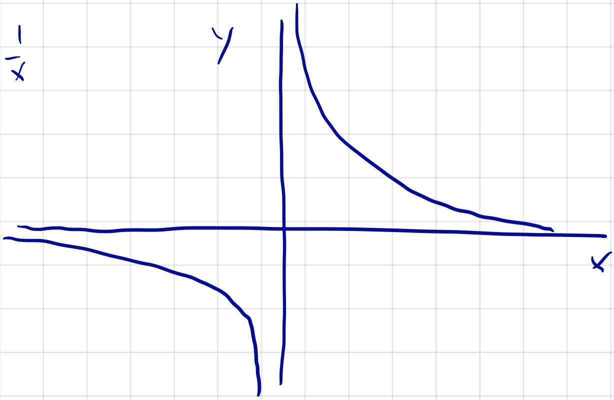


We'll say $\lim_{x \to 0} \frac{1}{x^2} = \infty$.

$-\infty$ is also possible

($\lim_{x \to a} f(x) = \infty$ if the values of f can be made as large (and positive) as you wish taking x close to a (but $x = a$ is not required))

$f(x) = \frac{1}{x}$



$$\lim_{x \to 0} \frac{1}{x} \text{ does not exist.}$$

But we have one-sided limits:

$$\lim_{x \to 0^+} \frac{1}{x} = \infty \quad \longleftarrow \text{ from the right}$$

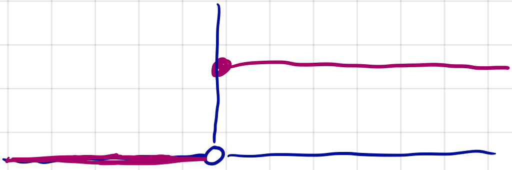
(only $x > 0$ is under consideration)

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty \quad \longleftarrow \text{ from the left}$$

We can also have one-sided limits in other contexts:

Heaviside function

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$



$$\lim_{x \to 0^+} H(x) = 1$$

$$\lim_{x \to 0^-} H(x) = 0$$

Important fact: if $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = M$

1) If $L = M$ $\lim_{x \to a} f(x) = L \; (= M)$

2) If $L \neq M$, $\lim_{x \to a} f(x)$ d.n.e.

About justifying infinite limits:

Consider $f(x) = \dfrac{5}{3-x}$.

What is $\lim_{x \to 3^+} f(x)$?

top: $\lim_{x \to 3^+} 5 = 5$

$\frac{5}{0}$ looks like it might be infinite. But what sign?

bottom $\lim_{x \to 3^+} 3-x = 0$

For x near 3, $x > 3$ $3-x < 0$.
E.g. $x = 3.01$ $3-x = -0.01$.

I'll indicate this by 0^-.

$\dfrac{5}{0^-} \Rightarrow -\infty$ (5 divided by a really small negative number is a large negative number)

$\dfrac{+}{0^+}, \dfrac{-}{0^-} \Rightarrow +\infty$

$\dfrac{+}{0^-} \dfrac{-}{0^+} \Rightarrow -\infty$

$\dfrac{0}{0^{\pm}} \to$ indeterminate.