Tues: Recitation worksheet

Wed 8/29:

Cover solutions to WS-1-1

Groupwork: WS 1-3

Thu 8/30

Volunteers: 1-3: 5, 7, 8, 10

Exponent Rules by examples

E.g.	Rule
$5^6 = 5\cdot5\cdot5\cdot5\cdot5\cdot5$ $= (5\cdot5\cdot5\cdot5)\cdot(5\cdot5)$ $= 5^4\cdot5^2$	$(r>0,\ a,b\in\mathbb{R})$ $r^{a+b} = r^a r^b$
$(5^2)^3 = 5^2\cdot5^2\cdot5^2$ $= 5^{2+2+2}$ $= 5^6$	$(r>0,\ a,b\in\mathbb{R})$ $(r^a)^b = r^{ab}$

E.g.	Rule
$(2\cdot 7)^3 = (2\cdot 7)(2\cdot 7)(2\cdot 7)$ $= (2\cdot 2\cdot 2)(7\cdot 7\cdot 7)$ $= 2^3\,7^3$	$(r, s > 0, \ a \in \mathbb{R})$ $(rs)^a = r^a s^a$
$5^1 = 5$	$(r>0)$ $r^1 = r$

Consequences: $r^0 = 1 \quad (r = r^1 = r^{1+0} = r^1 r^0 = r \cdot r^0)$

$r^{-1} = \frac{1}{r} \quad \left(1 = r^0 = r^{1+(-1)} = r^1 r^{-1} = r\, r^{-1} \Rightarrow r^{-1} = 1/r\right)$

Exponential function:

$$f(x) = r^x \qquad r > 0$$

(e.g. 2^x, 10^x e^x)

Do not confuse with power functions

$$p(x) = x^a \qquad a \in \mathbb{R}$$

e.g. x^2, x^3, $x^{1/3}$, x^{-1}.

the x here is downstairs
not upstairs

Exponential functions arise when there is an associated doubling (or halving) phenomenon.

E.g. A population of caribou grows by 10% per year and initially has 1000 animals at time $t = 0$ years.

Claim:

$$p(t) = 1000 \cdot (1.1)^t$$

($C r^{at}$ general form of an exponential function)

Did this work?

$p(0) = 1000\ (1.1)^0 = 1000$ ✓

$p(1) = 1000\ (1.1)^1 = 1100$ ✓

$p(2) = 1000\ (1.1)^2 = 1100 \cdot (1.1)$

$= 1100 + \left(\frac{1}{10}\right) 1100$ ✓

$= 1210$

How many caribou after 18 months = 1.5 years?

$p(1.5) = 1000\ (1.1)^{1.5} \approx 1153.69$

Where's the doubling?

Suppose instead our function was

$$p(t) = 1000 \, 2^t \qquad (1.1 \to 2)$$

$p(0) = 1000$
$p(1) = 2000$
$p(2) = 4000$
$p(3) = 8000$ etc.

The population doubles every time t goes up by $\underline{1}$.

How about

$$p(t) = 1000 \, 2^{-t} \; ?$$

$p(0) = 1000$
$p(1) = 500$
$p(2) = 250$

Now it halves every time t goes up by 1

How about

$$p(t) = 1000\ 2^{t/3}$$

$$p(0) = 1000$$
$$p(3) = 2000$$
$$p(6) = 1000\ 2^{6/3}$$
$$= 1000\ 2^2$$
$$= 4000$$

The population doubles when t goes up by 3.

$$p(t) = 1000\ 2^{-t/3}$$

Now it halves every time t goes up by 3

(we call 3 the half-life of the population,

the time it takes for the pop to be cut in half; see text examples concerning radioactive isotopes)

Back to the caribou:

General form Cr^{at} r, a are negotiable.

I claim $$1000\,(1.1)^t = 1000 \cdot 2^{at}$$

for a suitable choice of a.

$$2^{at} = (2^a)^t$$

So this is possible if $2^a = 1.1$,

in which case, the population will double every $1/a$ years

How do we solve for a? That's exactly what log's are for:

Recall

$$\log_{10}(10^x) = x \qquad 10^{\log_{10} y} = y$$

$$\log_2(2^x) = x \qquad 2^{\log_2 y} = y$$

etc.

Exponent rules have companion log rules

$$10^{x+y} = 10^x 10^y \longleftrightarrow \log_{10}(ab) = \log_{10}(a) + \log_{10}(b)$$

$$a = 10^x \quad b = 10^y$$

$$x = \log_{10} a \quad y = \log_{10} b$$

$$(10^x)^y = 10^{xy} \longleftrightarrow \log_{10}(a^b) = b\log_{10}(a)$$

$$\log_{10}(a^y) = xy$$

$$a = 10^x \quad y = b$$

$$x = \log_{10} a$$

$$10^0 = 1 \longleftrightarrow \log_{10}(1) = 0$$

You are welcome to use any base you want; they are all equally good, and one can be rewritten in terms of another.

Common: $\log_{10}$

$\log_e = \ln$

$\log$ → science, eng $\log_{10}$

$\log$ → math, matlab $= \ln$

I'll avoid plain log.

$$1.1 = 2^a$$

$$\log_{10}(1.1) = \log_{10}(2^a) = a \log_{10}(2).$$

$$a = \frac{\log_{10}(1.1)}{\log_{10}(2)} \approx 0.1375 \qquad \approx 7.27$$

$$\log_2(1.1) = \log_2(2^a) = a \log_2(2) = a.$$

$$\log_2(1.1) = \frac{\log_{10}(1.1)}{\log_{10}(2)} \approx 0.1375$$

In general, $\log_a(x) = \dfrac{\log_b(x)}{\log_b(a)}$

Population doubles every $\frac{1}{a} \simeq 7.27$ years.