The goal of this worksheet is for you construct an orthogonal projection onto a subspace.

Let

$$\mathbf{a}_1 = (1,1,1)$$
$$\mathbf{a}_2 = (0,1,0)$$

Let $V = \text{span}(\mathbf{a}_1, \mathbf{a}_2)$. We showed in class that the projection matrix is

$$P = A(A^tA)^{-1}A^t$$

for any matrix A where the columns of A are a basis for V.

1. Explain why the columns of $A = [\mathbf{a}_1, \mathbf{a}_2]$ form a basis for V.

2. Form the matrix A^tA. Then compute its inverse.

3. Compute the full projection matrix.

4. Use your projection matrix to project $(5,2,1)$ onto V

The fact is, we hate making inverse matrices, like $(A^tA)^{-1}$. In the 2×2 case, this is OK. Otherwise, it is a ton of work, and is typically not necessary. For projection, we use a three step process to project $\mathbf{b}$ onto V.

1. Form A^tA.
2. Solve $A^tA\mathbf{x} = A^t\mathbf{b}$. You'll use elimination. For a real problem you'll use LU factorization. For a real, real problem a computer will use LU factorization for you.
3. The projection is $A\mathbf{x}$.

5. Use elimination to solve $A^tA\mathbf{x} = A^t(5,2,1)$.

6. Compute $A\mathbf{x}$ and ensure your solution matches the solution you found earlier,.