F314 Linear Algebra: Quiz 9 November 3, 2017 Name:

Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 3 \\ -2 & 4 \end{bmatrix}$$

1. Determine a basis for the left null space of A.

$$A^T = \begin{bmatrix} 1 & 2 & 1 & -2 \\ 1 & 2 & 3 & 4 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & 1 & -2 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 2 & 1 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 2 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\boxed{n_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad n_2 = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 1 \end{bmatrix}}$$

2. Find a basis for $V^\perp$, where V is the span of $(1, 2, 1, -2)$ and $(1, 2, 3, 4)$.

Observe: $V = C(A)$ so $V^\perp = C(A)^\perp = N(A^T)$.

So n_1, n_2 from (1) form a basis for $V^\perp$.

3. Determine if there is a solution of $A\mathbf{x} = (1, 2, 7, 16)$. DO NOT BOTHER FINDING A SOLUTION (IF ONE EXISTS).

$b = (1, 2, 7, 16)$

$n_1 \cdot b = -2 + 2 = 0$

$n_2 \cdot b = 5 - 21 + 16 = 0$

So there is a solution.