F314 Linear Algebra: Quiz 8 October 27, 2017 Name:

1. Define what it means for vectors $\mathbf{v}_1, \dots, \mathbf{v}_n$ to be linearly independent.

If $c_1 v_1 + \cdots + c_n v_n = 0$, then $c_1 = 0$, $c_2 = 0, \dots, c_n = 0$.

2. Let W be a subspace of $\mathbb{R}^n$. Define what it means for vectors $\mathbf{v}_1, \dots, \mathbf{v}_n$ to be a basis for W.

They are a basis for W if

1) $W = \text{span}(v_1, \dots, v_n)$
2) the vectors are linearly independent.

3. Consider the matrices A and R below; I promise you that R can be obtained from A by elimination.

$$A = \begin{bmatrix} 5 & -1 & 13 & 0 & -3 & 9 \\ 5 & 5 & -5 & -6 & -13 & -3 \\ 2 & 0 & 4 & -2 & -6 & -8 \\ 3 & 3 & -3 & -4 & -5 & 7 \\ -5 & -2 & -4 & 4 & 8 & -4 \\ -1 & 1 & -5 & -2 & -2 & -6 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 4 \\ 0 & 1 & -3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find bases for the column spaces and the null spaces of A. Also find bases for the column spaces and the null spaces of R.

Column space of A: $(5, 5, 2, 3, -5, 1)$
$(-1, 5, 0, 3, -2, 1)$
$(0, -6, -2, -4, 4, -2)$
$(-3, -13, -6, -5, 8, -2)$

Column space of R: $(1, 0, 0, 0, 0, 0)$
$(0, 1, 0, 0, 0, 0)$
$(0, 0, 1, 0, 0, 0)$
$(0, 0, 0, 1, 0, 0)$

Nullspace of R and A: $(-2, 3, 1, 0, 0, 0)$
$(-4, 2, 0, 1, 3, 1)$

Recall

$$A = \begin{bmatrix} 5 & -1 & 13 & 0 & -3 & 9 \\ 5 & 5 & -5 & -6 & -13 & -3 \\ 2 & 0 & 4 & -2 & -6 & -8 \\ 3 & 3 & -3 & -4 & -5 & 7 \\ -5 & -2 & -4 & 4 & 8 & -4 \\ -1 & 1 & -5 & -2 & -2 & -6 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 4 \\ 0 & 1 & -3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4. Show (by finding a concrete linear combination) that the columns of A are not linearly independent.

$$-2 \begin{bmatrix} 5 \\ 5 \\ 2 \\ 3 \\ -5 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 5 \\ 0 \\ 3 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 13 \\ -5 \\ 4 \\ -3 \\ -4 \\ -5 \end{bmatrix} = 0$$

5. True or false: the rows of A are linearly independent. Justify your answer.

They are linearly dependant. The zero rows of R show that some rows add to zero with nonzero coefficients.