Name:

1. If

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

what is A? Then solve $A\mathbf{x} = (0, 1, 0)$.

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 4 & 4 & 2 \\ 6 & 16 & 5 \end{bmatrix}$$

① Solve $L\vec{c} = \vec{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$c_1 = 0$

$2c_1 + c_2 = 1 \Rightarrow c_2 = 1$

$3c_1 + 4c_2 + c_3 = 0 \Rightarrow c_3 = -4c_2 - 3c_1 = -4$

$$\vec{c} = \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}$$

② Solve $U\vec{x} = \vec{c}$

$2x_3 = c_3 = -4 \Rightarrow x_3 = -2$

$4x_2 = c_2 = 1 \Rightarrow x_2 = 1/4$

$2x_1 + x_3 = 0 \Rightarrow x_1 = -x_3/2 = 1$

$$\vec{x} = \begin{bmatrix} 1 \\ 1/4 \\ -2 \end{bmatrix}$$