

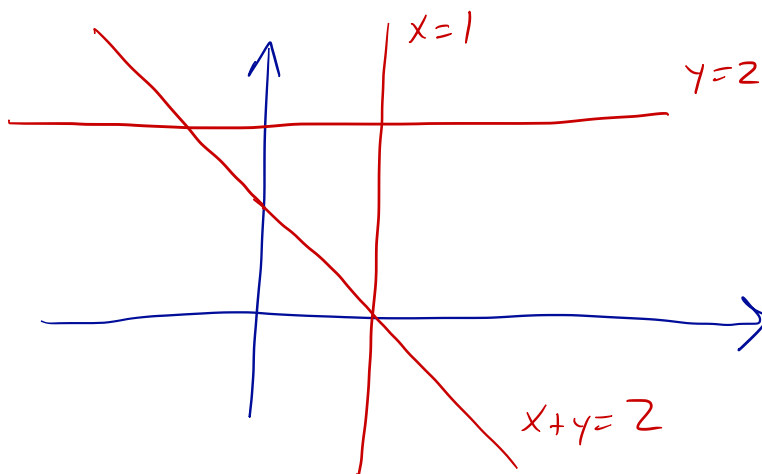
**Name:** Consider the problem to solve for  $(x, y)$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

1. Express this problem from the column perspective by filling in the blanks:

Find a linear combination of  $(1, 0, 1)$  and  $(0, 1, 1)$  that equals  $(1, 2, 1)$ .

2. Draw a diagram that expresses this problem from the row perspective.



$$\begin{aligned} x &= 1 \\ y &= 2 \\ x + y &= 1 \end{aligned}$$

3. Find a vector  $\mathbf{n}$  that is perpendicular to each of the columns of the matrix in this equation.

$$\mathbf{n} = (1, 1, -1). \quad \text{Check: } \begin{aligned} u &= (1, 0, 1) & \mathbf{n} \cdot u &= 1 \cdot 1 + 1 \cdot 0 + (-1) \cdot 1 = 0 \\ v &= (0, 1, 1) & \mathbf{n} \cdot v &= 1 \cdot 0 + 1 \cdot 1 + (-1) \cdot 1 = 0 \end{aligned}$$

4. All of the linear combinations of the columns of this matrix lie in a certain plane. What plane is it?

The plane with normal  $\mathbf{n} = (1, 1, -1)$  passing through  $\vec{0}$ .

5. Use  $\mathbf{n}$  to show that this problem does not have a solution.

$$\mathbf{n} \cdot \mathbf{b} = (1, 1, -1) \cdot (1, 2, 1) = 1 + 2 - 1 = 2 \neq 0.$$