Name:

1. Consider the vector $\mathbf{u} = (1,1,1)$. Find two **nonzero** vectors $\mathbf{v}$ and $\mathbf{w}$ that are perpendicular to $\mathbf{u}$ and are perpendicular to each other.

$$\vec{v} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \qquad \vec{w} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\vec{u} \cdot \vec{w} = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot (-2) = 0$$

$$\vec{u} \cdot \vec{v} = 1 \cdot 1 + (-1) \cdot 1 + 1 \cdot 0 = 0$$

$$\vec{v} \cdot \vec{w} = 1 \cdot 1 + (-1)(1) + 0 \cdot (-2) = 0$$

2. Again, $\mathbf{u} = (1,1,1)$. Suppose $\mathbf{z}$ is another vector and $||\mathbf{z}|| = 2$.

Note: $||u|| = (1^2 + 1^2 + 1^2)^{1/2} = \sqrt{3}$

1. What is the maximum possible value of $\mathbf{u} \cdot \mathbf{z}$? $2\sqrt{3}$ (Cauchy–Schwarz)
2. What is the minmum possible value of $\mathbf{u} \cdot \mathbf{z}$? $-2\sqrt{3}$ (ditto)
3. What is the maximum possible value of $||\mathbf{u} + \mathbf{z}||$? $2 + \sqrt{3}$ (triangle ineq).
4. Give an example of a vector $\mathbf{z}$ with $||\mathbf{z}|| = 2$ such that $||\mathbf{u} + \mathbf{z}||$ attains its maximum possible value.

$$z = \frac{2}{\sqrt{3}} \vec{u} = \left(\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}} \right).$$

$$u + z = \vec{u} + \frac{2}{\sqrt{3}} \vec{u} = \left(1 + \frac{2}{\sqrt{3}}\right) \vec{u}. \quad ||\vec{u} + \vec{z}|| = \left(1 + \frac{2}{\sqrt{3}}\right)\sqrt{3} = 2 + \sqrt{3}.$$