Name:

1. Consider the vectors

$$\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}.$$

Show that the vector $(2, 0)$ can be written as a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

$$a\begin{bmatrix} 3 \\ -2 \end{bmatrix} + b\begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \rightarrow$$

$$\begin{aligned} 3a + 6b &= 2 \\ -2a + 4b &= 0 \end{aligned} \longrightarrow \begin{cases} a + 2b = \frac{2}{3} \\ -2a + 4b = 0 \end{cases}$$

$$\rightarrow 8b = \frac{4}{3}, \; b = \frac{1}{6}$$

$$a + 2b = \frac{2}{3}, \; b = \frac{1}{6} \Rightarrow a = \frac{1}{3}$$

$$\boxed{a = \frac{1}{3}, \; b = \frac{1}{6}}$$

2. Consider the vectors

$$\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}.$$

Either show that $\mathbf{x}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$, or explain why it is not.

If $a\vec{v} + b\vec{w} = \vec{x}$,

$$\begin{bmatrix} a + b \\ -a + b \\ a + b \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

so $a + b = 2$, $a + b = 4$ ← impossible!

So $\vec{x}$ is <u>not</u> a linear combination of $\vec{v}$ and $\vec{w}$.

3. In the diagram below, draw the set of all linear combinations $a\mathbf{v} + b\mathbf{w}$ such that $a + b = 1/2$.