F314 Linear Algebra: Quiz 12 December 1, 2017 Name:

1. Compute the eigenvalues of

oops!

$$A = \begin{bmatrix} \cancel{6}\ 4 & 2 \\ -3 & -3 \end{bmatrix}.$$

For each eigenvalue, compute an eigenvector. For full credit you must clearly put a box around each eigenvalue/eigenvector pair.

$$\det(A-\lambda I) = \det\begin{bmatrix} 4-\lambda & 2 \\ -3 & -3-\lambda \end{bmatrix} = -(4-\lambda)(3+\lambda)+6$$
$$= -(-\lambda^2+\lambda+12)+6$$
$$= \lambda^2-\lambda-6$$
$$= (\lambda-3)(\lambda+2)$$

eigenvalues: $\lambda = 3$, $\lambda = -2$

$A-3I = \begin{bmatrix} 1 & 2 \\ -3 & -6 \end{bmatrix}$ $x_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ in null space (easy to spot!)

$A+2I = \begin{bmatrix} 6 & 2 \\ -3 & -1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ in null space (easy to spot!)

$\boxed{\lambda_1 = 3 \quad x_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}}$ $\boxed{\lambda_2 = -2 \quad x_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}}$

2. Show that $\mathbf{x} = (-1, 2)$ is an eigenvector of

$$A = \begin{bmatrix} 3 & 2 \\ -2 & -2 \end{bmatrix}$$

with eigenvalue -1. Then compute $A^{73}\mathbf{x}$.

$$Ax = \begin{bmatrix} 3 & 2 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3+4 \\ 2-4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \underbrace{-}_{\lambda=-1}\underbrace{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}_{x}$$

$$A^2x = A(Ax) = A(-x) = -Ax = (-1)^2x$$
$$A^3x = A(A^2x) = A(-1)^2x = (-1)^3x$$
$$\vdots$$
$$\boxed{A^{73}x = (-1)^{73}x = -x.}$$