Here are some things you should know for the midterm, which will cover all of chapters 1 and 2. Everything from the homework and the quizzes is, in particular, fair game. You must know all of that. Here are more study ideas.

Given vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, what is a linear combination of these vectors?

Describe all the linear combinations of $(1,1)$ and $(0,1)$.

Describe the set of all vectors $s(1,1) + t(-1,2)$ such that $s > 0$ and $t < 0$.

What is the definition of $\mathbf{v} \cdot \mathbf{w}$?

What is the definition of the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$?

When is $\mathbf{v}$ perpendicular to $\mathbf{w}$.

If $\mathbf{v}$ and $\mathbf{w}$ are unit vectors, what is the geometric interpretation of $\mathbf{v} \cdot \mathbf{w}$?

How is the length of a vector associated with the dot product?

What is the Cauchy-Schwartz inequality?

What is the triangle inequality?

Given two vectors in $\mathbb{R}^5$, how do you compute the angle between them?

If $\mathbf{v}$ has length 1 and $\mathbf{w}$ has length 2, what are the longest and shortest $\mathbf{v} + \mathbf{w}$ can be?

If $A = [\mathbf{v}_1, \cdots, \mathbf{v}_n]$ and $\mathbf{x} = (x_1, \ldots, x_n)$, how is matrix multiplication $A\mathbf{x}$ defined?

Express the following problem using matrix multiplication: Find a linear combination of $(1,0,1)$, $(2,2,2)$, and $(7,0,7)$ that equals $(9,0,7)$.

Express the following problem using matrix multiplication: Find the intersection of the planes $z = 1$, $x + y - 2z = 4$, and $2x + 2y + 1 = 0$.

Use elimination to convert a linear system to an equivalent upper-triangular linear system.

Use back-substitution to solve an upper-triangular linear system.

Given an augmented matrix $[A\mathbf{b}]$, find a matrix B such that $B[A\mathbf{b}]$ is an upper-triangular system.

Know elimination matrices and row-exchange (i.e. transposition) matrices like the back of your hand. If E is an elimination matrix, know how to quickly multiply EA.

What happens if you multiply a matrix by an elimination matrix on the **right**? I.e. what is AE? If P is a permutation matrix that interchanges rows 3 and 5, what is AP?

If A is a matrix and $B = [\mathbf{v}_1, \ldots, \mathbf{v}_n]$, what is AB? Use this definition to compute

$$\begin{bmatrix} 4 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 1 & 4 \end{bmatrix}$$

Express the third column of

$$AB = \begin{bmatrix} 4 & 1 & 6 & -1 \\ 2 & -1 & 4 & 4 \\ 1 & 6 & -1 & 2 \\ 7 & 2 & -3 & 6 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & -4 \\ 4 & 9 & 6 & 3 \\ 1 & 6 & -1 & 2 \\ 4 & 9 & 3 & 7 \end{bmatrix}$$

as a linear combination of the columns of A.

When is it legal to multiply $A\mathbf{x}$? When is it legal to multiply AB?

What are the row, column, and row-column perspectives on matrix multiplication?

Know how to divide matrices into blocks to be able to do block multiplication.

Given

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 1 \\ -1 & -1 & 3 \\ 2 & 0 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 1 \\ 9 & 0 & 7 \\ 4 & -2 & 5 \end{bmatrix}$$

and

$$AB = \begin{bmatrix} * & ? & ? \\ * & ? & ? \\ * & * & * \\ * & * & * \end{bmatrix}$$

compute the subblock marked with ? marks by dividing A and B into appropriately sized blocks, and doing block multiplication to compute just the subblock.

If $R_1, \ldots, R_n$ are the rows of A, used block multiplication to compute AB.

Be able to show that the following are true:

- If A is invertible, the only solution of $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$.
- If there is a non-zero solution $\mathbf{x}$ of $A\mathbf{x} = \mathbf{0}$, then A is not invertible.
- If A is invertible, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.
- If the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution, then A is not invertible.
- If A is invertible, then the equation $A\mathbf{x} = \mathbf{b}$ can have at most one solution.
- If $\mathbf{x}_1$ and $\mathbf{x}_2$ are two different solutions of $A\mathbf{x} = \mathbf{b}$, then A is not invertible.

How is the inverse of a matrix defined?

If A is a 7×7 matrix and $\mathbf{w}$ is the 4^{th} column of A^{-1}, what equation does $\mathbf{w}$ solve? If R is the second row of A^{-1}, what is RA?

Know how to compute a matrix inverse using Gauss-Jordan elimination.

Know how to factor $A = LU$.

Given a factorization $A = LU$, be able to solve $A\mathbf{x} = \mathbf{b}$.

What is the transpose of a matrix?

$\mathbf{y} \cdot (A\mathbf{x}) = ? \cdot \mathbf{x}$

Given the description of a permutation matrix, write it down. I.e. write down the 4×4 permutation matrix that takes row 1 to row 3, row 3 to row 4, and row 4 to row 2.

Given a permutation matrix, find its inverse.