**Exercise 1:** Suppose we have two points $\mathbf{p}_1 = (x_1, y_1)$ and $\mathbf{p}_2 = (x_2, y_2)$ and we would like to find the equation of a line $y = mx + b$ going through those points. Substitute these points into the equation of the line to obtain two equations for the unknowns $m$ and $b$.

**Exercise 2:** Suppose $\mathbf{p}_1 = (0.2, 0.7)$ and $\mathbf{p}_2 = (0.8, -0.4)$. What are the equations for $m$ and $b$?

**Exercise 3:** Write these equations in matrix form:

$$A \begin{bmatrix} m \\ b \end{bmatrix} = \mathbf{v}.$$

Explicitly write down what $A$ and $\mathbf{v}$ are.

**Exercise 4:** Solve these equations as follows. First, enter the matrix $A$ into Octave. Then enter the right-hand side $\mathbf{v}$. Form the augmented matrix $A_{aug}$ by entering `Aaug=[A,v]`. Solve the system using `M=rref(Aaug)`. Then extract $m$ and $b$ from their locations in $M$.

**Exercise 5:** Verify your solution is correct by making the a plot that contains your line (using the values of $m$ and $b$ that you computed), and contains the two points plotted as circles.

**Exercise 6:** Now find the equation of a parabola $y = ax^2 + bx + c$ passing through the points $(-1, 1.5)$, $(3, 32.2)$, $(5, -42.6)$. You must

a) Record the system of equations to solve.

b) Convert the system into a matrix system.

c) Solve the system using Octave (record the command you used and the solution).

d) Generate a plot that contains the parabola and the three points.

**Exercise 7:** Explain, in terms of intersecting planes, why if you have only two data points, these determine infinitely many parabolas.

**Exercise 8:** Explain, in terms of intersecting planes, why if you have four data points, you expect there will be no parabola containing all four points.

**Exercise 9:** If you have 7 data points $(x_n, y_n)$, with all of the $x_n$'s different, these uniquely determine a polynomial of some order. What is the order?

**Exercise 10:** Find the equation of a polynomial of the kind discussed in the previous step that passes through the points $(1, 4)$, $(2, -1)$, $(4, 7)$, $(5, -3)$, $(8, 1)$, $(9, -10)$ $(11, 3)$. You will find the following Ocatave command to be helpful: If $\mathbf{c} = (c_1, c_2, \ldots, c_n)$ has been entered into MATLAB as the column vector `c`, then the command `vander(c)` will return the matrix

$$\begin{bmatrix} c_1^{n-1} & c_1^{n-2} & \ldots & c_1 & 1 \\ c_2^{n-1} & c_2^{n-2} & \ldots & c_2 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_n^{n-1} & c_n^{n-2} & \ldots & c_n & 1 \end{bmatrix}.$$

This is called a *Vandermonde matrix*.) Record the formula for your polynomial, together with a graph of it and the data points.