1. Consider the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}.$$

a) Use elimination to find a matrix E_{21} such that $E_{21}A = U$, where U is upper triangular.

b) Compute E_{21}^{-1}.

c) Explain why $A = E_{21}^{-1}U$.

d) Observe that E_{21}^{-1} is lower-triangluar; let's call it L, so $A = LU$. Use L and U to solve

$$A\mathbf{x} = (5, 7).$$

Recall: first solve $L\mathbf{c} = \mathbf{b}$, then $U\mathbf{x} = \mathbf{c}$.

2. Consider the matrix

$$A = \begin{pmatrix} 4 & 1 & -1 \\ -8 & 0 & -1 \\ 12 & 7 & -8 \end{pmatrix}$$

a) Compute the matrices E_{21}, E_{31} and E_{32} obtained using elimination so that

$$E_{32}E_{31}E_{21}A = U$$

where U is upper triangular.

b) Using properties of inverse matrices from page 84 (The Inverse of a Product) explain why

$$A = E_{21}^{-1}E_{31}^{-1}E_{12}^{-1}U.$$

c) Write down the individual matrices E_{21}^{-1}, E_{31}^{-1}, and E_{12}^{-1}.

d) Use brute force matrix multiplication to compute

$$L = E_{21}^{-1}E_{31}^{-1}E_{12}^{-1}.$$

Is L lower-triangular?

e) Use $A = LU$ to solve $A\mathbf{x} = (7, -26, -1)$.