1. Consider the matrix

$$A = \begin{pmatrix} 2 & 4 & 5 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 0 & 8 & 12 \end{pmatrix}$$

a) Find a collection of vectors $\mathbf{w}_k$ such that $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{w}_k \cdot \mathbf{b} = 0$ for each k.

b) Determine if $A\mathbf{x} = (5, 4, 6, 4)$ has a solution. You do not need to find the solution, if it exists.

2. Find a basis for the orthogonal complement of the plane in $\mathbb{R}^4$ spanned by $(4, 6, 7, 8)$ and $(5, 10, 11, 12)$.

3. For a square matrix, explain why its null space and its left null space have the same dimension. Then answer the following: is it possible that a square matrix A has an inverse but A^T does not?

4. Suppose $\mathbf{v}$ is a non-zero vector in $\mathbb{R}^4$ and let $A = \mathbf{v}\mathbf{v}^T$. The questions below concern an arbitrary choice of $\mathbf{v}$. Still, to get a picture of what is going on, you might find it helpful to pick a random vector $\mathbf{v}$ in $\mathbb{R}^4$ and examine the properties of that particular $A = \mathbf{v}\mathbf{v}^T$. Nevertheless, your answers need to work in general, not just for one choice of $\mathbf{v}$.

- Compute the dimension of the column space of A. You should explain your answer in terms of the column perspective of matrix multiplication.
- Compute the dimension of the null space of A.
- Explain why the left null space equals the null space for this matrix.
- True or false: $A\mathbf{x} = \mathbf{0}$ if and only if $\mathbf{x}$ is perpendicular to $\mathbf{v}$.
- Does the column space equal the row space for this matrix?
- Find a condition on $\mathbf{v}$ that ensures $A^2 = A$.

5.